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Rotary ultrasonic machining of single-crystal silicon

Kh M Asif Raihan[1], Isaac Obembe[1], Deven Wieland[2], Christopher Jones[3] and Meng Zhang[1,*]

[1] Department of Industrial and Manufacturing Systems Engineering, Kansas State University, Manhattan, USA
[2] John Deere, Coffeyville, USA
[3] Department of Civil Engineering, Kansas State University, Manhattan, USA

* Correspondence author; E-mail: meng@ksu.edu.

1. Calculation for the maximum pressure during RUM

Highest force $F_H = 81.57$ N (Experiment Table 5)
Lowest force $F_L = 57.56$ N (Experiment Table 5)
Tool inner diameter $d_i = 10$ mm
Tool outer diameter $d_o = 12$ mm
Area $A_0 = \frac{\pi}{4}\left({d_o}^2 - d_i^2\right) = 35.558\ \text{mm}^2$
Abrasive concertation $C_a = 100$ [1]
Abrasive material density $\rho = 3.52 \times 10^{-3}\frac{g}{mm^3}$ [1]
Grit/Mesh size = 100–120 [1]
Abrasive size $S_a = 0.137$ mm [1]
Abrasive edge radius = 1–10 µm [2]
Diamond elastic modulus E = 1200 Gpa [3]
Silicon elastic modulus : 130 Gpa [4]
Poisson's ratio for diamond = 0.0691 ~ 0.2 [5]
Poisson's ratio for silicon = 0.26 [4]
Composite $\frac{1}{E^*} = \frac{1-{\vartheta_d}^2}{E_d} + \frac{1-{\vartheta_s}^2}{E_s}$
$E^* = 124.97$ Gpa
Number of active abrasives on the edge of a RUM cutting tool

$$N = \left(\frac{(C_a \times 0.88^{-3})}{\left(100 \times \left(\frac{\pi}{6}\right) \times \rho \times {S_a}^3\right)}\right)^{\frac{2}{3}} \times A_0 \ [6]$$

N = 79.43 ~ 80

Peak pressure $P0 = \frac{3}{2}\left(\frac{16WE^{*2}}{9\pi^3NR^2}\right)^{\frac{1}{3}}$ Hertzian contact [7]

For highest load, $P0 = \frac{3}{2}\left(\frac{16 \times 81.57 \times (124.97 \times 10^9)^2}{9\pi^3 \times 80 \times (10 \times 10^{-6})^2}\right)^{\frac{1}{3}} = 31.35\ GPa$, R = 10µm

For highest load, $P0 = \frac{3}{2}\left(\frac{16\times81.57\times(124.97\times10^{9})^{2}}{9\pi^{3}\times80\times(1\times10^{-6})^{2}}\right)^{\frac{1}{3}} = 145.52\ GPa$, R = 1 μm

For lowest load, $P0 = \frac{3}{2}\left(\frac{16\times57.56\times(124.97\times10^{9})^{2}}{9\pi^{3}\times80\times(10\times10^{-6})^{2}}\right)^{\frac{1}{3}} = 27.91\ GPa$, R = 10μm

For lowest load, $P0 = \frac{3}{2}\left(\frac{16\times57.56\times(124.97\times10^{9})^{2}}{9\pi^{3}\times80\times(1\times10^{-6})^{2}}\right)^{\frac{1}{3}} = 129.55\ GPa$, R = 1 μm

Table S1. Calculated maximum pressure during RUM.

Cutting force (N)	Abrasive radius ((μm)	Peak pressure (GPa)
57.56	10	31.35
57.56	1	145.52
81.57	10	27.91
81.57	1	129.55

Therefore, with rotary ultrasonic machining (RUM) cutting tool the local peak pressure was more than the pressure induced phase transformation (Si-I → Si-II), requirement for creating ductile deformation zones [8].

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