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# On anomalous radioactive decay according to the energy metrics formalism in the Deformed Space-Time (DST) theory

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† We dedicate this article to Giovanni Cherubini, who greatly contributed to the investigations on the subject and, among other things, conceived the "Cherubini proposal" cited here.

## Highlights:

- **Observation of Anomalous Decay in Co-57 Under Ultrasound**: The study analyzes time-dependent anomalies in the 14.4 keV Fe-57 emission line induced by Co-57 decay under 2.25 MHz ultrasound, identifying a deviation from the standard exponential decay pattern.
- **Application of Deformed Space-Time (DST) Theory**: The anomalous behavior is interpreted using DST theory, which posits that under certain energy conditions, nuclear interactions occur in a non-Minkowskian, deformed metric, violating Local Lorentz Invariance (LLI).
- **Ultrasound as a Catalyst for DST Reactions**: The study finds that only ~4.3 nanoseconds—1/100 of an ultrasound cycle—are sufficient to trigger space-time deformation, enabling DST nuclear reactions that transform radioactive nuclei into stable forms.
- **Two Distinct Decay Mechanisms**: The decay process is attributed to a combination of standard weak interaction (natural decay) and DST-induced strong interaction (anomalous transformation), with the latter occurring in microcavities (Ridolfi cavities) acting as nuclear micro-reactors.
- **Evidence for Latency and Metric Overlap Effects**: The persistence of deformation effects beyond the brief sonication interval and the presence of multimetric overlap suggest hysteresis and mimicry effects, providing a theoretical foundation for cross-interaction energy influence (e.g., "Mignani mimicry").

**Abstract:** We analyze the data reported in the literature on the time variation of the 14.4 keV Fe-57 line induced by the emission of Co-57. An anomalous decay of Co-57 was observed in the presence of ultrasound sonication. In this study, we investigate this behavior within the framework of Deformed Space-Time (DST) theory and deduce that only a fraction of an ultrasound cycle is sufficient to produce this anomaly.

**Keywords:** Deformed Space-Time; radiative decay; energy metrics formalism; ultrasound sonication; anomalous decay of Co-57

## 1. Introduction

Phonon-nuclear coupling was suggested [1] as a mechanism able to convert the high energy of vibrational nuclear states into many vibrational states of lower energy, via a many-phonon exchange. To check this hypothesis, the emission lines of Fe-57 excited by radioisotope Co-57 were investigated during 2.25 MHz ultrasound irradiation (sonication) [2].

Some Fe-57 emission lines exhibited an unexpected time-dependent behavior, corresponding to an anomalous decay of Co-57. In the following, we focus on the 14.4 keV line (Figure 1), which is the most intense vibrational transition, though the method can also be applied to other cases where anomalous behavior is detected. It should be noted that Figure 1 is an adaptation by the present authors of data privately communicated in advance by the authors of [2] and subsequently published, albeit in a different form, in the same reference.

Further details on the experimental setup, measurement precision, error bars associated with the experimental data points, and other methodological aspects can be found in [2], along with the details of the experimental results.

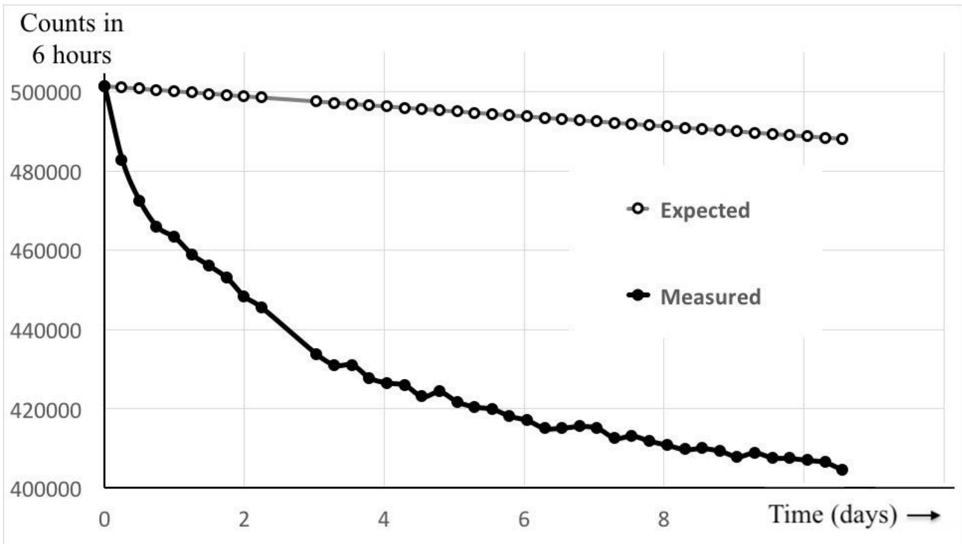


**Figure 1.** Intensity of the 14.41 keV line of Fe-57 as a function of time: Experimental behaviour with sonication (black circles) and expected values corresponding to the Co-57 decay constant of 271.74 days (empty circles). This Figure is adapted from the data of [2].

Our analysis will be performed by following the Deformed Space-Time (DST) theory see e.g. [3].

According to this theory, when nuclear DST reactions occur, the hadronic interactions operate in a deformed space-time, which is characterized by a not minkowskian-flat metric. According to the DST theory, we are just experimenting here the effect of DST reactions which are a new, "anomalous" kind of reactions. These DST reactions change the nuclei from the parent one to a son nucleus having different nuclear weight, as represented in the following schematic process:

PARENT NUCLEUS + ELECTRON → SON NUCLEUS + ELECTRON NEUTRINO.

In this framework a radioactive nucleus can be turned into a "neutral" one, or rather a nucleus not radioactive at all. Therefore, any change, or rather any activity reduction is due to the lack of radioactive nuclei, and not to any kind of acceleration of the decay.

In fact, within the DST framework, different metrics were deduced for the different interactions, see e.g. [3].

The generalized interval $ds^2$ is given by:

$$ds^2 \equiv b^2{}_0\, c^2\, dt^2 - b^2{}_1\, dx_1{}^2 - b^2{}_2\, dx_2{}^2 - b^2{}_3\, dx_3{}^2 \quad (1)$$

where the metric parameters $b_i$ (that in the Minkowski space-time are all equal to 1) depend on the energy E of the process and on the type of interaction.

Values of the metric parameters different from one correspond to a violation of the Local Lorentz Invariance (LLI).

In the case of strong interaction under the DST formalism, the metric parameters $b_0$ (time parameter), $b_1$, $b_2$ and $b_3$ (space parameters) were deduced, see e.g. [3], starting from the data of the pion pair production in the experiment performed in 1984 by the UA1 collaboration at CERN [4].

$$b_0{}^2 = b_3{}^2 = b^2(E)$$

$$b^2(E) = 1 \rightarrow \text{if } E < E_{0\,\text{strong}} = 367.\,5 \pm 0.\,4 \text{ GeV} \quad (2)$$

$$b^2(E) = (E/\,E_{0\,\text{strong}})^2 \qquad \text{if } E > E_{0\,\text{strong}}$$

$$b_1{}^2 = 2/25$$

$$b_2{}^2 = 4/25$$

A feature of DST theory is the role of energy which, besides being a parameter, is a fifth coordinate. Thus, a five-dimension (5D) space is considered, where the above reported 4D-space-time is a sub-space. We clarify here that only the $E > E_{0\,\text{strong}}$ metric parameter violates Local Lorentz Invariance (LLI), as the deformed space-time where the hadronic interactions operate is characterized by a not minkowskian-flat metric.

A challenging question is the so-called "multimetric", *i.e.* a particle can live in different spacetime for different interactions. In our case there can be an overlapping of both leptonic and hadronic metrics analogous to the overlapping between wave functions in quantum mechanics, as indeed already observed e.g. in [3], for the case of the electromagnetic metric between the deformed space of the weak interaction and the deformed space-time of the strong interactions when they act both competitively and cooperatively on the same physical entity which is the cobalt nucleus. In the cooperative case the metrics can mimic coherent nuclear phenomena.

In recent times, the radioisotope Ni-63 was shown [5], to behave differently from the natural decay if sonicated by 35 KHz ultrasounds with 15 W of transferred power. In fact, the bremsstrahlung intensity of the emitted beta-electrons decreased by 14 ± 6% after 200 seconds of sonication.

This attenuation was attributed to DST nuclear reactions, which reduced the number of radionuclides by transforming them into other nuclei [5]. After this sudden decrease, the decay process was supposed to proceed with a non-modified time constant.

A similar effect can operate in the above cited case of Co-57 [2] (we shall call Hagelstein experiment) if the ultrasounds and the whole experimental apparatus accidentally produced conditions able to induce a space-time deformation, with the consequent occurrence of DST nuclear reactions.

## 2. Data analysis

We propose that, in analogy with the case of Ni-63, two different mechanisms operated in the Co-57 experiment: DST nuclear transformations which are ruled by the hadronic (also called strong) nuclear interaction metric and exponential decay, which is ruled by the DST leptonic (or weak) nuclear interaction metric. Needless to say, the anomalies are catalysed by the ultrasounds, but not caused by them, rather by the nuclear reactions due to the DST metrics of interactions.

In order to find the value of time coordinate corresponding to the violation of LLI for the hadronic interaction in the decay Co-57 → Fe-57, the speed along the energy coordinate, *i.e.* the power W, must be evaluated and, more in particular, if it reaches the critical value [3]

$$W_{0\,\mathrm{strong}} = 4.8 \cdot 10^{30}\ \mathrm{eV/s}$$

The power, energy and time intervals can assume different values in the space-time of strong interactions, where the reactions occur, and in the space-time of our instrumentation, which is the electro-magnetic one. When a reacting particle is detected, the energy E in the two space-times, *i.e.* flat and non-flat (or deformed), is the same but the time intervals are different.

The condition for a hadronic DST reaction is

$$W \geq W_{0\,\mathrm{strong}}$$

From the above given metric parameters one can deduce, if $E \geq E_{0\,\mathrm{strong}}$:

$$b_0^2(E) = (E/ E_{0\,\mathrm{strong}})^2 = dt_{\mathrm{strong}}^2/dt^2 \rightarrow dt_{\mathrm{strong}}/dt = E / E_{0\,\mathrm{strong}} \tag{3}$$

where t and $t_{\mathrm{strong}}$ are the electromagnetic and hadronic times, respectively.

Thus, for a short time lapse:

$$E = E_{0\,\mathrm{strong}}\, t_{\mathrm{strong}} /t \tag{4}$$

and the power (energy speed) is

$$W = E /t = E_{0\,\mathrm{strong}}\, t_{\mathrm{strong}} /t^2 \tag{5}$$

Our proposal is that the anomaly verified in the Hagelstein experiment corresponds to the part with not minkowskian-flat metric of the strong interaction.

If the conditions for space-time deformation are accidentally satisfied in a part of the sample, we aim to evaluate the time t = X necessary for the deformation to take place in other parts of the sample.

We work out the experimental data obtained in the Hagelstein experiment above reported in Figure 1. As a first step, we evaluate the "time law" of emission along the fifth coordinate, which is energy.

To this aim, we consider that each emission corresponds to an amount of 14.41 keV of energy and that the first measurement (corresponding to the initial time in Figure 1) was performed at a later time with respect to the beginning of ultrasound irradiation.

The results are reported in Figure 2, which is obtained applying Equation (6) to the data displayed in Figure 1.

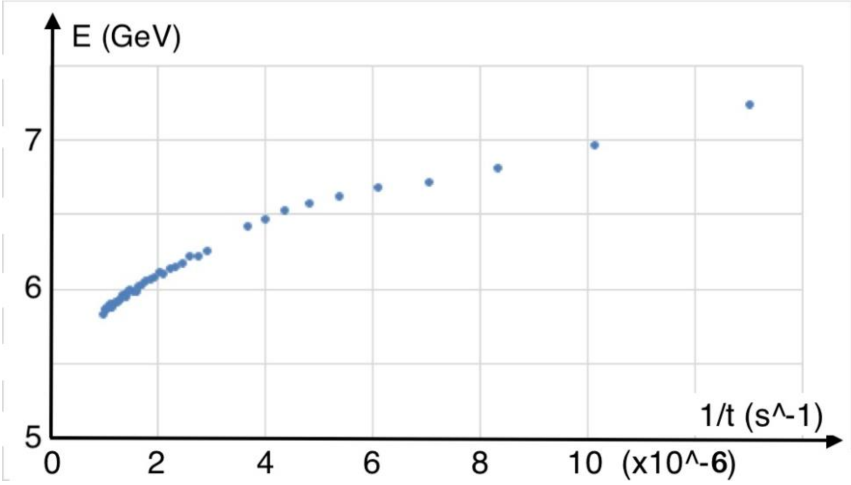


**Figure 2.** Energy from 14.41 keV photons as a function of 1/t (t being the time in the electromagnetic space-time).

Starting from formula (3) we obtain the following time law of energy in the deformed space-time of hadronic interaction, when the threshold energy $E_{0,strong}$ that separates the flat time space of Minkowski from the deformed space for the hadron metric is passed:

$$E(t) = E_{0,strong.} \cdot \frac{t_{strong}}{t} \tag{6}$$

Given that this threshold is exceeded in a part of the sample, here we evaluate the time t = X necessary for further deforming the space-time, thus violating the LLI for the hadronic interaction inside the decay Co-57 → Fe-57.

The speed along the energy coordinate must reach the above reported critical value $W_{0\ strong}$.

With reference to the equation (5) we now determine a limit time t = X and the corresponding $t_{strong}$ = TS near the deformation threshold such that W is greater than or equal to $W_{strong}$.

$$W \geq W_{strong}$$

$$E_{0,strong} \times \frac{t_{strong}}{t^2} \geq W_{strong} \tag{7}$$

So that t=X, the upper limit is defined as

$$X = \left(TS \text{ x } \frac{E_{0,strong}}{W_{strong}}\right)^{1/2} \tag{8}$$

$$= (TS)^{1/2} * 2.8 \cdot 10^{-10}$$

In Equation (8) we have used the values formerly introduced for E and W. Following Equation (6), TS can be derived from the slope M of the straight line reported in Figure 3, whose value is obtained from this figure. It reports the three data of Figure 2 corresponding to the shortest values of t (*i.e.* the largest values of 1/t):

$$TS = M/E_{0strong} = 242 \text{ s}$$

As a consequence, using Equation (8) we can calculate the upper limit of t, yielding the value X = 4.3 ns. It is interesting to notice that this upper limit corresponds to just about 1/100 of ultrasonic period, supplied at 2.25 MHz frequency in the Hagelstein experiment in [1].

In this way, we deduce that the power transferred by the ultrasounds to the Co-57 → Fe-57 system through the metal plate supplies the energy to deform the space-time for the hadron interactions in a time interval X = 4.3 ns.

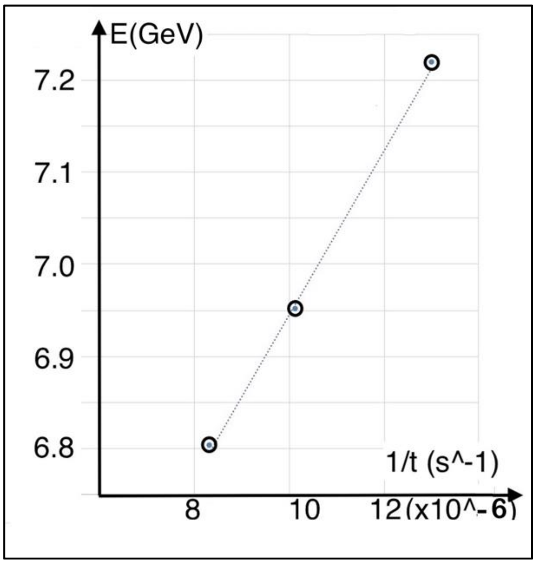


**Figure 3.** Energy from 14.41 keV photons as a function of 1/t (t being the time in the electromagnetic space-time) in the first times of emission. The reported values are those of Figure 2 at higher values of 1/t. The angular coefficient of the best fit straight line is $M = 8.89 \cdot 10^4$ GeV·s.

This deformation is not stable due to the critical density (in spatial volume) which is related to the time. Stability is maintained until the conditions of critical density in space and time are satisfied.

In fact, our evaluation is valid close to the thresholds for energy and power. If the accumulated energy increases with time, the ratio $dt_{strong}/dt$ increases too (Equation (3)), thus giving a negative contribution to the slope of the curve E *vs.* 1/t (Figure 2).

The above considerations provide a rough estimate of the limit time X, which is extremely short and, at first glance, appears incompatible with the timescales observed in the phenomenon recorded by Hagelstein.

However, it is possible to hypothesize the existence of a latency (or hysteresis) mechanism that extends the perturbation effects on space-time well beyond the time interval X. In this regard, we note that latency had been reported since the first experiments with ultrasound applied to ferrous materials and consequent neutron emissions (e.g. in [5]). Delays of some fractions of hour were observed with respect to the times of ultrasound application.

Moreover, the non-continuous application of the ultrasounds in the Hagelstein experiment can account, albeit to a lesser extent, for further anomalies in the decay curve.

In our evaluation, only the energy from the main line of Fe-57 is considered. If the contributions from other lines are not negligible and they add up, then the time t = X is still smaller. In fact, a higher energy E implies a higher $t_{strong}/t$ ratio in Equation (6) and this fact implies a lower value of t in Equation (7).

Looking at Figure 1, it is evident that a strong decrease in counts occurs at the beginning, while progressively smaller variations are registered over time.

This behavior cannot be attributed solely to the decreasing number of active cobalt nuclei. The emissions at the longest times remain about 80% of the initial ones, meaning that the decrease in active nuclei is approximately 20%. However, the slope of the curve varies significantly more than 20%. Specifically, the decrease in counts per 6 hours is about 50,000 in the first two days, whereas it drops to approximately 5,000 in the last two days—indicating a much greater variation than 20%.

Thus, in addition to the decreasing number of active nuclei, the effectiveness of space-time deformation also diminishes at longer times. A similar behaviour was also observed in an anomalous decay of Ni-63 nuclei (see [5]). It was explained by considering that DST nuclear reactions only occur in micro-reactors, that are micrometric bubbles already present in the solution. If new reactors do not form, further deformation and reactions cannot take place.

In a similar way, in the case of solid material, products of DST nuclear reactions were only observed inside those cavities (the Ridolfi cavities) having size of some microns (See [6]): these cavities acted as micro-reactors.

In analogy with the latter case, we presume that microcavities already present in the solid material acted as micro-reactors in the Hagelstein experiment. Since no new cavities could form, the deformation and the reactions gradually diminished over time.

In conclusion, two main mechanisms contribute to the observed decay: the strong nuclear interaction, which is responsible for the anomalous decay inside the Ridolfi cavities, and the weak nuclear interaction, which governs the natural decay process.

In analogy with the results obtained for Ni-63, nuclei not belonging to the known decay chain of cobalt are expected to be generated in the former case.

A further point to consider: it is well known that the energy provided by one interaction (e.g., electromagnetic) can influence the effects of a different interaction (e.g., nuclear). However, within the framework of DST theory, the connection between different interactions may be more pronounced. One key reason is that the energy and power of a given interaction can correspond to threshold values of another interaction, thereby inducing space-time deformation in the latter. A further and weird connection is the so-called "Mignani mimicry effect", which was also recently mentioned [3]: the space-time deformation of an interaction assumes the same characteristics of a different interaction, thus inducing the effects of the latter (see [5]).

This last effect too can give its contribution when this new kind of reactions, the DST-reactions, take place.

## 3. Discussion

In the literature [1,2,7], it has been suggested that phonon-nuclear interaction plays a significant role in explaining the phenomena addressed in the present work. However, this interpretation does not seem plausible to us, given that nuclear energy scales are approximately one hundred thousand times greater than atomic energy scales (MeV *versus* Rydberg) and that nuclei are about one hundred thousand times smaller than atoms (Fermi *versus* Angstrom). Any applied pressure, including sound waves at any frequency, primarily affects atoms rather than nuclei. Consequently, direct energy transfer between atomic vibrations and nuclear states appears highly unlikely. Hence, we disagree with any models proposing an interaction phonon-nucleus for the evident disparity of the size and energy between them, *i.e.* phonon and nucleus. After all, the phonon-nucleus interaction is a merely phenomenological model meant to interpret measurements, rather than extract fundamental physics. On the contrary, we attribute alterations in nuclear reactions to the energy densification of the DST theory [8], as reactions where it is the deformation of spacetime that absorbs the excess of energy, thus preserving total energy conservation. From this point of view the decay is not accelerated, otherwise there would a large and well-detectable increase of the activity, *i.e.* an increasing in the counting, as well as in the counting rate.

That being said, it is conceivable that the appropriate energy density, as predicted by DST theory, could still be achieved through random pressure fluctuations, effectively mimicking a phonon-nucleus interaction that would otherwise be extremely difficult to realize. From this perspective, a technology based on a purely phonon-nuclear phenomenological model would be challenging to develop and control, making it difficult to establish a reliable and reproducible approach for practical applications.

The DST theory, as explained in the manuscript, is an extension of general relativity (see e.g. [3]), where it is also explained how the Finzi's solidarity principle was used in formulating the new theory. Moreover, classical explanations have been sought by us, in the form of possible deviations of the radioactive decay from exponential law [9], which is well known in the classical literature, see e.g. the textbook in [10]. However, in this case such deviations are not viable in such great amount as observed and reported in [2], hence the explanation based on the conventional mechanism does not apply, at least not for such amount, *i.e.* nearly 20% change.

As long as the decay time of Co-57 is concerned, the weak interaction is operating. However, we assume that the observed anomalous decay is mainly to be attributed to the strong interaction, which transforms Co nuclei into different ones. Thus, the instantaneous event of decay, induced by weak interaction, is not affected by the space-time deformation.

This is the main effect we consider. Other effects, such as time deformation which could affect the decay time (weak interaction), are not the focus of our considerations.

We remark that the effect of space-time deformation on Co nuclei is considered. The gamma decay of Fe-57 is only a probe to investigate the Co behaviour. We also notice that the observed anomalous decay is not to be attributed to a phonon-nuclear coupling, as suggested by the authors of [1,2], but rather to effects they did not control.

## 4. Conclusions and outlook

Our research group has conducted studies on altering radioactive decay rates of isotopes such as Ni-63 and Th-228 using cavitation techniques. In these experiments, cavitation—a process involving the formation, growth, and implosive collapse of bubbles in a liquid—was applied to solutions containing these isotopes, leading to significant reductions in their radioactivity. The theoretical framework underpinning these observations is the DST theory, which posits that under certain conditions, space-time can be deformed, leading to violations of LLI. This deformation is characterized by specific metric parameters that describe the extent and nature of the space-time distortion.

However, explicit metric parameters within the DST framework for Ni-63 and Th-228 have not been fully detailed. While our group has extensively discussed the theoretical aspects of DST and its implications for nuclear processes, the specific metric parameters corresponding to the cavitation-induced decay of these isotopes are not provided in the published works.

Starting from the results obtained by our working group for the decay of Ni-63 [5,6] and Th-228 [5] it is possible to demonstrate that similar situations occurred, perhaps randomly and for short periods of time, in the Hagelstein experiment [2]. The exposition is based on the relationship between the time in the deformed space-time of hadronic interaction and the time in electromagnetic space-time, following the prediction of the DST theory, see e.g. [3].

The results presented by Hagelstein for the Co-57 → Fe-57 decay would therefore constitute a further experimental test (separate and distinct) confirming the results and conclusions already presented by our team in the case of Ni-63 and Th-228.

It would be thus confirmed, more generally, that it is possible to alter the radioactive decays by imposing critical parameter values, corresponding to metrics for interactions in a deformed space-time (DST), in which Lorentz invariance (LLI) is violated. This implies another type of violation related to the speed of light and the causality principle [11]. Experimental tests of the possible violations of the LLI were proposed earlier [12,13] some of which are planned in space [14,15].

Hereby, we mention, in passing, that it could be of interest to perform the following test, *i.e.* measuring whether radioactivity increases during sonication (because more nuclei decay) or not (because nuclei undergo a DST reaction and turn into others, without emission from decay). In the second case, Hagelstein's thesis would be no less than disproved. As a side remark, we wish to remind that the case of an isotropic space-time in an isotropic Universe has been treated in its dynamics, from the point of view of the mathematical foundations and the numerical analysis, in the book by S. Benenti [16].

Recently, the asymmetry observed in various experiments has been linked to the asymmetry of the Cosmic Microwave Background Radiation, suggesting the existence of a fundamental asymmetry in interactions. This perspective could provide new insights into the longstanding question of symmetry breaking in the history of the Universe [17].

As a future outlook, we propose conducting radioactivity measurements during acoustic stress. If the authors of [2] are correct and the decay constant indeed varies—specifically, if the decay time decreases—then a corresponding sudden increase in radiation emission per unit time (activity) should be observed. If no such increase is detected, the hypothesis would not hold. We refer to this as the Cherubini proposal.

## Data availability

The datasets generated during and/or analysed during the current study are available from the corresponding author on reasonable request.


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## Authors' contribution

Conceptualization, SB and FC; methodology, SB and FC; software, SB and FC; validation, SB and FC..; formal analysis, SB and FC; investigation, SB and FC; resources, SB and FC; data curation, SB and FC; writing—original draft preparation, SB and FC; writing—review and editing, SB and FC; visualization, SB and FC; supervision, SB and FC; project administration, SB and FC; funding acquisition, SB and FC. All authors have read and agreed to the published version of the manuscript.

## Conflicts of interests

The authors declare no conflict of interest.

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