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Mean-variance tradeoff of bitcoin inverse futures

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Abstract: Bitcoin inverse futures are dominant derivative contracts traded in the cryptocurrency market. We aim to understand the mean-variance tradeoff of such contracts through quantitative studies. To this purpose, we derive explicit representations for the expectation and variance of the returns on Bitcoin inverse futures and obtain their first-order approximations. The empirical findings show that Bitcoin inverse futures are more (resp. less) risky than standard futures when the market is in backwardation (resp. contango). We further find that Bitcoin inverse futures bear higher downside risk, as measured by semi-deviation, than standard futures.

Keywords: bitcoin; downside risk; inverse futures; volatility

1. Introduction

Bitcoin (BTC) is the first decentralized cryptocurrency and has the largest market capitalization and trading volumes among thousands of cryptocurrencies. The strong research interest in Bitcoin is evidenced by a growing number of literature; see [1–4] for a short list. We refer readers to [5–7] for survey articles on Bitcoin and other cryptocurrencies, and the related works that are not reviewed here to save length. The key purpose of this paper is to investigate the mean-variance tradeoff of Bitcoin futures.

The Chicago Mercantile Exchange (CME) and the Chicago Board Options Exchange (CBOE) launched Bitcoin futures for trading on December 10, 2017 and December 18, 2017, respectively.[1] Both CME and CBOE Bitcoin futures are *standard* contracts, which treat Bitcoin as the underlying and use fiat currency (U.S. dollar, USD with symbol $) as the denomination and settlement currency. Recent studies on Bitcoin standard futures traded on CME and CBOE focus on their role in price discovery, market efficiency, and hedging performance; see, e.g., [8–14]. In comparison, Bitcoin futures offered by many online exchanges (e.g., BitMEX and OKEx) are *inverse* contracts, which use fiat cryptocurrency (such as USDT, 1 USDT $\approx$ 1 USD) as the underlying and BTC as the denomination and settlement unit. Please see Tables A.1 and A.2 in [15] for contract details of Bitcoin standard and inverse futures.

Although both standard and inverse futures contracts co-exist in the Bitcoin markets, their market capitalization and trading volumes differ significantly. As seen from Figure 1, the daily trading volumes of CME standard Bitcoin futures are almost negligible, less than

[1] Note that CBOE had ceased offering Bitcoin futures after June 19, 2019, leaving CME the only major venue for trading Bitcoin standard futures.

8% of Binance during the observation period (from March 23 to April 23 in 2020).[2] Despite having the dominant market shares in the Bitcoin futures market, Bitcoin inverse futures are not well studied in the current literature, with some exceptions. [16] use the perpetual inverse contracts from BitMEX to investigate price discovery, informational efficiency, and hedging effectiveness. [15] study the minimum variance hedging problem of Bitcoin inverse futures to spot markets and the hedging effectiveness of the optimal strategy. [17] formulate an optimal investment problem for a utility maximizing agent who invests in Bitcoin spot and inverse futures. [18] analyze two unique features of inverse futures, automatic liquidation and leverage selection, and their role in hedging Bitcoin spot markets. We remark that the success of Bitcoin inverse futures likely motivates the design of Bitcoin inverse options, which have been gaining significant attention in recent years (see [19–21]).

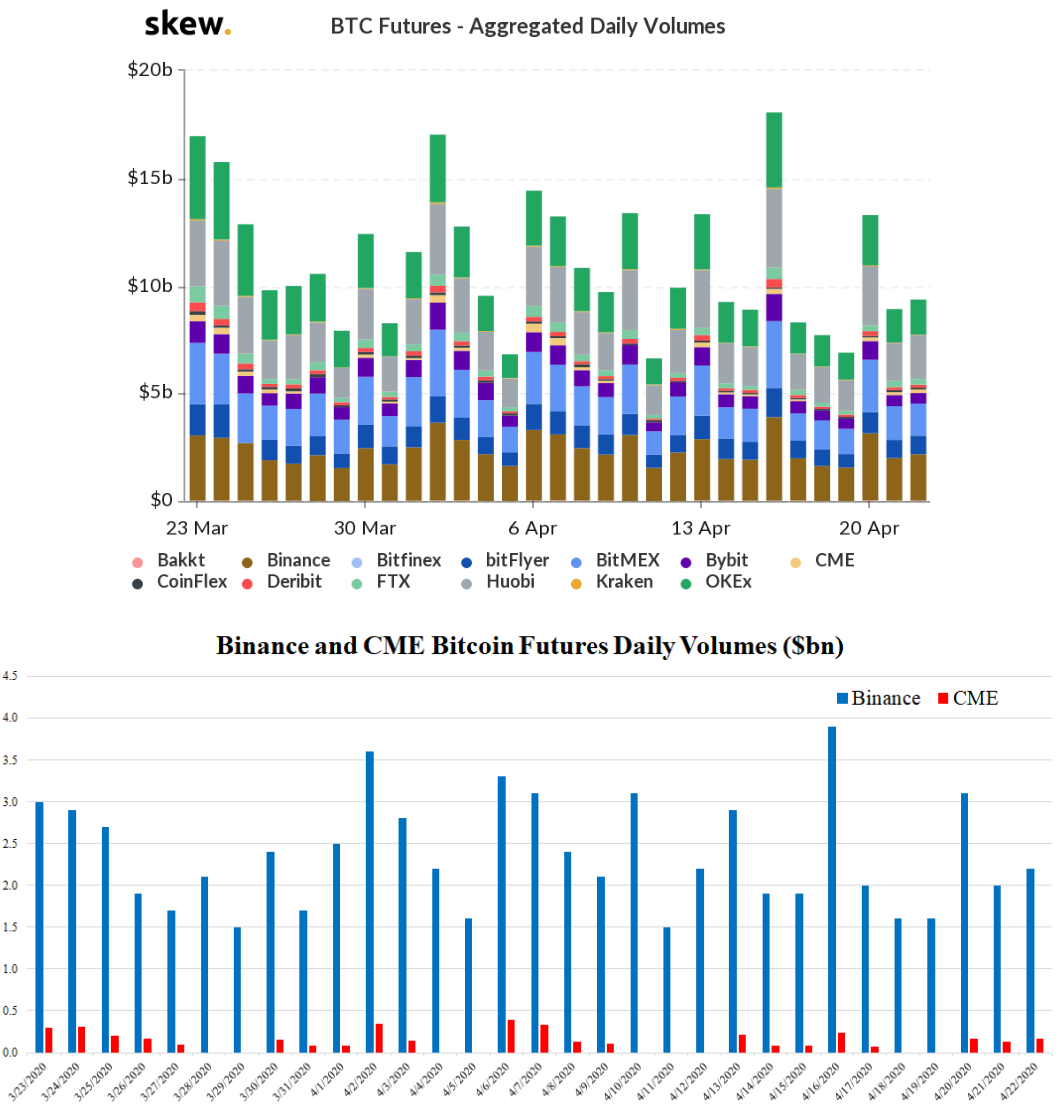

Figure 1. Bitcoin futures daily trading volume.

Note. The upper panel plots the aggregated daily trading volumes of Bitcoin futures at major exchanges from March 23 to April 23 in 2020. The CME is the only venue for trading standard futures contracts, while all others trade inverse futures contracts. In the lower panel, to gain better view on the comparison, we draw the charts for the Binance and CME only. The y-axis in both graphs is in unit $bn (billion USD). Data source are from https://www.skew.com/dashboard/bitcoin-futures.

[2] This finding is more universal across different time periods for several major exchanges, e.g., BitMEX, OKEx, and Binance.

As is well known, a primary use of futures is to hedge the spot market, and an optimal hedge is often defined as the one that minimizes the variance of the portfolio consisting of spot asset and its futures (see [22]). When a standard futures contract is used to construct the hedging portfolio, the analysis is straightforward because the payoff structure of standard futures is in a simple, linear form as seen in (2.1). However, if the hedger trades inverse futures, the analysis is far from being trivial because the payoff structure of inverse futures is intrinsically *nonlinear* as given by (2.2). In consequence, the first step towards understanding inverse futures is to analyze their expected returns and variance (volatility) resulting from the unique payoff structure in (2.2). To the best of our knowledge, the mean-variance (risk-return) tradeoff of Bitcoin inverse futures has not been studied before in the existing literature. Note that although several papers study the hedging with Bitcoin inverse futures (see, e.g., [15] and [18]), they treat the futures returns as an exogenous process (time series) and construct an optimal hedge that depends on the mean and variance of the returns, which they estimate directly from data. However, as already argued, such a topic deserves a detailed investigation given the enormous trading volumes of inverse futures (see Figure 1) and given its importance in the optimal hedging study and portfolio management. Therefore, in this paper, we aim to fully decipher the mean-variance tradeoff of Bitcoin inverse futures and compare their riskiness with the counterpart standard futures. The first objective of this paper is to conduct an analytical study on the expected return and variance of the Bitcoin inverse futures based on the "raw data" (i.e., Bitcoin spot and futures reference prices). The second objective is to compare Bitcoin standard and inverse futures, with a particular focus on their riskiness, measured by the volatility of their returns.

Our main contributions are in three aspects. First, we derive exact representations of the mean and variance of the returns of Bitcoin inverse futures, shown in (2.6) and (2.8), respectively. These results are new to the cryptocurrency literature and pin down the precise dependence of the performance of inverse futures on the spot price S and the futures reference price F. Since the exact representations in (2.6) and (2.8) involve summations of infinite sequences, we further obtain their first-order approximations in (2.13) and (2.14), which are easy to understand. To assess the accuracy of such approximations, we conduct an empirical study and find that they are as accurate as the sample mean and standard deviation. Second, we provide a in-depth comparison on Bitcoin standard and inverse futures. The key finding is that inverse futures are more risky than standard futures when the market is in backwardation (resp. contango). (Recall that if the Bitcoin spot price S is greater the futures reference price F, the market is in backwardation, and the opposite scenario is called contango.) Third, Bitcoin inverse futures bear higher downside risk, as measured by semi-deviation, than standard futures. This effect is induced by the inverse payoff structure since it is more sensitive to the decreasing of prices. We remark that these findings are immediately beneficial to those who have difficult deciding which futures contracts to trade, since our results decode the riskiness involved in the trading of Bitcoin standard and inverse futures.

The rest of the paper is organized as follows. In Section 2, we present our main theoretical results on representing the mean and variance of futures returns and their approximations. In Section 3, we conduct empirical studies to investigate the accuracy of the volatility approximations and (de)leverage effect. Our concluding remarks are given in Section 4.

2. Main results

Throughout the paper, we denote by $\mathbb{E}$, $\mathbb{V}\text{ar}$, σ, and $\mathbb{C}\text{ov}$, the expectation, variance, standard deviation (volatility), and covariance operators respectively, under the physical probability measure $\mathbb{P}$. Let $S = (S_t)_{t\geq 0}$ and $F = (F_t)_{t\geq 0}$ represent the Bitcoin spot and futures reference prices, where both S and F are denominated in fiat currency, which is taken to be USD. We use ΔS and ΔF to denote the changes of the spot price S and the futures price F, respectively. For instance, let Δt be a time increment, $\Delta S_t = S_{t+\Delta t} - S_t$ and $\Delta F_t = F_{t+\Delta t} - F_t$ for all $t \geq 0$.

Our first task is to understand the unique payoff structure of Bitcoin inverse futures, which is significantly different from that of Bitcoin standard futures. To that end, consider a Bitcoin standard futures contract with notional value 1 BTC and a Bitcoin inverse futures contract with notional value K USD ($K > 0$), both linked to the same futures reference price F. If an investor longs one unit standard futures contract at time t and closes her position at $t+\Delta t$, then her payoff is given by

$$\text{Payoff (standard futures)} = F_{t+\Delta t} - F_t. \tag{2.1}$$

However, if the investor longs one unit *inverse* futures contract at time t and closes her position at $t+\Delta t$, then her payoff is given by

$$\text{Payoff (inverse futures)} = \frac{K}{F_t} - \frac{K}{F_{t+\Delta t}}. \tag{2.2}$$

Note that if the futures price F increases (i.e., $F_{t+\Delta t} > F_t$), a long position in both standard and inverse futures leads to positive profits. The above result shows that the payoff function of an inverse futures contract is *non-linear*, which brings extra risk factors and asymmetry effect into the mean and variance of the returns.

We define $F^B = (F_t^B)_{t\geq 0}$ by $F_t^B = \frac{K}{F_t}$, and interpret F_t^B as the *nominal value* of Bitcoin inverse futures per contract, denominated in BTC, at time t. Hereinafter, we use superscript $\cdot^B$ on a random variable (process) to emphasize that such a random variable (process) is denominated in BTC. Let us consider a *unit* long position in Bitcoin inverse futures, which is initiated at time t and closed at time $t+\Delta t$. We define (nominal) returns on Bitcoin futures as follows:

$$R_F := \frac{F_{t+\Delta t} - F_t}{F_t} = \frac{\Delta F_t}{F_t}, \tag{2.3}$$

$$R := \frac{\left(F_t^B - F_{t+\Delta t}^B\right) S_{t+\Delta t}}{K} = \left(\frac{1}{F_t} - \frac{1}{F_{t+\Delta t}}\right) S_{t+\Delta t}, \tag{2.4}$$

$$\widetilde{R} := \frac{S_{t+\Delta t} - S_t}{F_t} = \frac{\Delta S_t}{F_t}. \tag{2.5}$$

R_F defined in (2.3) is the (nominal) return on standard futures; see [22] for similar definition. R defined in (2.4) is the return on Bitcoin inverse futures, converted into the denomination of USD using the spot price. We interpret $\widetilde{R}$ in (2.5) as the *mixed* spot-futures return, which is useful in establishing the relationship between R and R_F in the subsequent analysis.

Our main focus is to investigate the time t conditional expectation (notation $\mathbb{E}_t$) and variance (notation $\mathbb{V}\text{ar}_t$) of Bitcoin inverse futures' return R. Note that we treat the mean of return as a return (performance) measure and the variance (or volatility) as a risk measure of Bitcoin inverse futures. The non-linear payoff feature brings higher-order risk factors into R. Proposition 2.1 below delivers this message.

Proposition 2.1. *We obtain the time t conditional expectation and variance of R by*

$$\mathbb{E}_t(R) = \frac{S_t}{F_t}\sum_{i=0}^{\infty}(-1)^i\,\mathbb{E}_t(R_F^{1+i}) + \sum_{i=0}^{\infty}(-1)^i\,\mathbb{E}_t\left[\widetilde{R}\cdot R_F^{1+i}\right], \tag{2.6}$$

$$\mathbb{V}\text{ar}_t(R) = \frac{S_t^2}{F_t^2}\sum_{i,j=0}^{\infty}(-1)^{i+j}\,\mathbb{C}\text{ov}_t\left(R_F^{1+i}, R_F^{1+j}\right) + \frac{S_t}{F_t}\sum_{i,j=0}^{\infty}(-1)^{i+j}\,\mathbb{C}\text{ov}_t\left(R_F^{1+i}, \widetilde{R}\cdot R_F^{1+j}\right) \tag{2.7}$$

$$+\sum_{i,j=0}^{\infty}(-1)^{i+j}\,\mathbb{C}\text{ov}_t\left(\widetilde{R}\cdot R_F^{1+i}, \widetilde{R}\cdot R_F^{1+j}\right). \tag{2.8}$$

Proof. From the definition of R in (2.4), we derive

$$R = \left(\frac{1}{F_t} - \frac{1}{F_{t+\Delta t}}\right) S_{t+\Delta t} = \frac{\Delta F_t}{F_t} \frac{S_{t+\Delta t}}{F_{t+\Delta t}} = R_F \frac{S_t + \Delta S_t}{F_t + \Delta F_t} \tag{2.9}$$

$$= R_F \frac{S_t/F_t + \Delta S_t/F_t}{1 + \Delta F_t/F_t} = R_F \frac{S_t/F_t + \widetilde{R}}{1 + R_F} \tag{2.10}$$

$$= R_F \left(\frac{S_t}{F_t} + \widetilde{R}\right) \sum_{i=0}^{\infty} (-1)^i R_F^i \tag{2.11}$$

$$= \frac{S_t}{F_t} \sum_{i=0}^{\infty} (-1)^i R_F^{1+i} + \sum_{i=0}^{\infty} (-1)^i \widetilde{R} R_F^{1+i}, \tag{2.12}$$

which naturally implies the results in (2.6) and (2.8) by taking condition expectation and conditional variance at time t. The proof is now complete.

The results in Proposition 2.1 are, despite explicit, complex to compute in practice and not easy for economic interpretation, since the sums of infinite series(s) are involved. Hence, to make them more applicable, we take the terms with index $i, j = 0$ in equations (2.6)-(2.8) and obtain the first-order approximations by

$$\mathbb{E}_t(R) \simeq \frac{S_t}{F_t} \mathbb{E}_t(R_F) + \mathbb{E}_t\left[\widetilde{R} \cdot R_F\right] := \widehat{\mathbb{E}}_t(R), \tag{2.13}$$

$$\mathbb{V}\text{ar}_t(R) \simeq \frac{S_t^2}{F_t^2} \mathbb{V}\text{ar}_t(R_F) + 2\frac{S_t}{F_t} \mathbb{C}\text{ov}_t(R_F, \widetilde{R} \cdot R_F) + \mathbb{V}\text{ar}_t(\widetilde{R} \cdot R_F) := \widehat{\mathbb{V}\text{ar}}_t(R). \tag{2.14}$$

In the later empirical studies, we show that the above first-order approximation $\sqrt{\widehat{\mathbb{V}\text{ar}}_t(R)}$ is as accurate as the sample standard deviation of R.

The results of Proposition 2.1 and the approximations (2.13)-(2.14) shed light on the complexity of the risk and return of Bitcoin inverse futures. First, the expected return of R is impacted by the returns of R_F and $\widetilde{R}$, i.e., by both the Bitcoin spot price S and futures price F. Second, the variance of R_F and the higher-order covariance between R_F and $\widetilde{R}$ both contribute to the risk (variance) of inverse futures' returns, and these factors are intertwined. This observation explains why perfect hedging is nearly impossible in practice for Bitcoin inverse futures. In comparison, if we consider the returns on Bitcoin standard futures R_F, it is clear that both $\mathbb{E}[R_F]$ and $\mathbb{V}\text{ar}[R_F]$ only depend on the futures price F. Third, there is a *volatility amplification* (*reduction*) effect on $\sigma_t(R)$, where $\sigma_t(R) = \sqrt{\mathbb{V}\text{ar}_t(R)}$. Here, volatility amplification (resp. reduction) effect refers to the situation when the inverse futures' intrinsic risk, as measured by $\sigma_t(R)$, is inflated above (resp. deflated below) the standard futures' risk (the volatility of the quoted futures price), as measured by $\sigma_t(R_F) = \sqrt{\mathbb{V}\text{ar}_t(R_F)}$. When the market is in contango, i.e., when futures price $F >$ spot price S (resp. backwardation, i.e., $F < S$), the volatility $\sigma_t(R)$ is deflated (resp. inflated) by the ratio S/F, when compared to the volatility $\sigma_t(R_F)$.

3. Empirical studies

3.1. Data description

In the following empirical study, we consider quarterly Bitcoin inverse futures traded on OKEx,[3] which is the most liquid contract on OKEx, accounting for nearly 85% of the trading volumes. We obtain the futures and spot price data spanning from 2018/10/07 to 2019/07/25 through its provided application programming interface (API) on www.okex.com. All price

[3] OKEx is the second largest exchange of Bitcoin futures by trading volumes, only trailing BitMEX. However, about 97% of the Bitcoin futures traded on BitMEX are perpetual contracts, which [16] call perpetual swaps.

data are in *daily* frequency and sampled at the Coordinated Universal Time (UTC).

We plot the Bitcoin spot price S and futures price F in Figure 2. The figure shows that S and F are highly positively correlated. We also report the summary statistics of the spot price change ΔS and futures price change ΔF in Table 1. An immediate observation is that both Bitcoin spot and futures are highly volatile, with the coefficient of variation for daily price changes over 30.

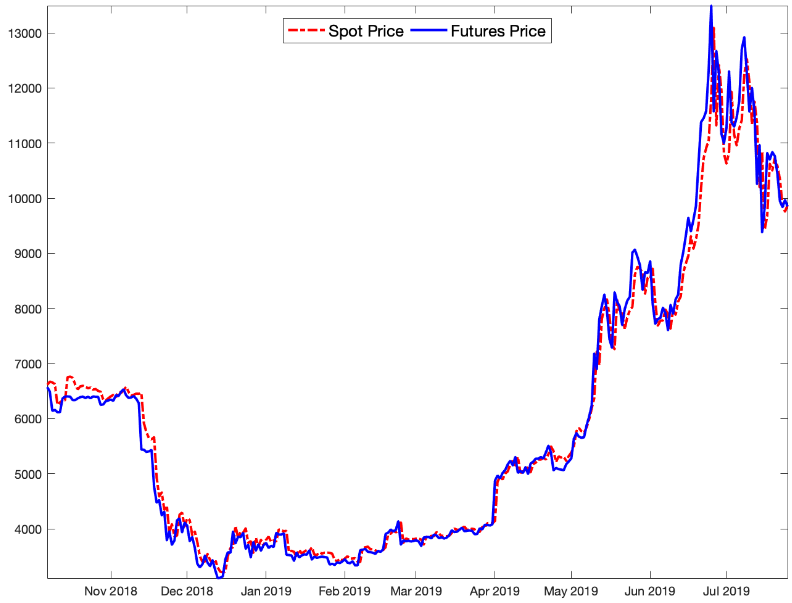

Figure 2. Bitcoin spot and futures prices on OKEx.

Note. The graph plots the Bitcoin spot and futures prices, in the unit of 1$ USD on the y-axis, on exchange OKEx from 2018/10/07 to 2019/07/25 (shown in x-axis).

Table 1. Summary statistics of daily price changes.

Variables	ΔS	ΔF	R_F	$\widetilde{R}$	R
Min	-1776.30	-1921.48	-0.14	-0.13	-0.15
P25%	-60.05	-63.33	-0.01	-0.01	-0.01
Median	8.70	9.36	0.00	0.00	0.00
Mean	10.99	11.34	0.0023	0.0020	0.0021
P75%	90.70	91.24	0.02	0.02	0.02
Max	1267.90	1162.35	0.19	0.17	0.19
S.D.	307.07	330.40	4.28%	3.97%	4.30%
Skewness	-0.57	-0.73	0.15	0.16	0.09
Kurtosis	11.42	11.16	6.31	6.15	6.34
Count	291	291	291	291	291

Notes. ΔS and ΔF are daily price changes of Bitcoin spot and futures. P25% and P75% refer to the 25% quantile and 75% quantile. The settlement reference price of Bitcoin futures on OKEx is the weighted average of the last Bitcoin prices in USD on major exchanges. S.D. stands for standard deviation.

The key futures returns are R_F (standard futures), R (inverse futures), and $\widetilde{R}$ (mixed spot-futures), which are defined in (2.3)-(2.5). Their summary statistics are given in Table 1. During the full sample period, the worst return for standard and inverse futures in USD (i.e., R_F and R) is about -15% and the best is about 19%, with daily volatility around 4.3% (annualized volatility over 80%). Given that the median of R_F and R is near zero, the probability of making profits from trading standard or inverse futures in a day is 50%, which is close to the daily profit probability of Bitcoin (53.69%) found in [23].

3.2. Accuracy of volatility approximations

In this subsection, we focus on the volatility (standard deviation) of Bitcoin inverse futures' returns R. Recall $\mathbb{V}\text{ar}_t(R)$ is given by (2.8), and its first-order approximation $\widehat{\mathbb{V}\text{ar}_t}(R)$ by (2.14). Let us denote by σ the volatility of arandom variable and $\widehat{\sigma}$ the first-order approximation of σ, e.g., $\sigma(R) = \sqrt{\mathbb{V}\text{ar}_t(R)}$ and $\widehat{\sigma}(R) = \sqrt{\widehat{\mathbb{V}\text{ar}_t}(R)}$. We investigate the accuracy of the first-order approximation $\widehat{\sigma}(R)$ to $\sigma(R)$.

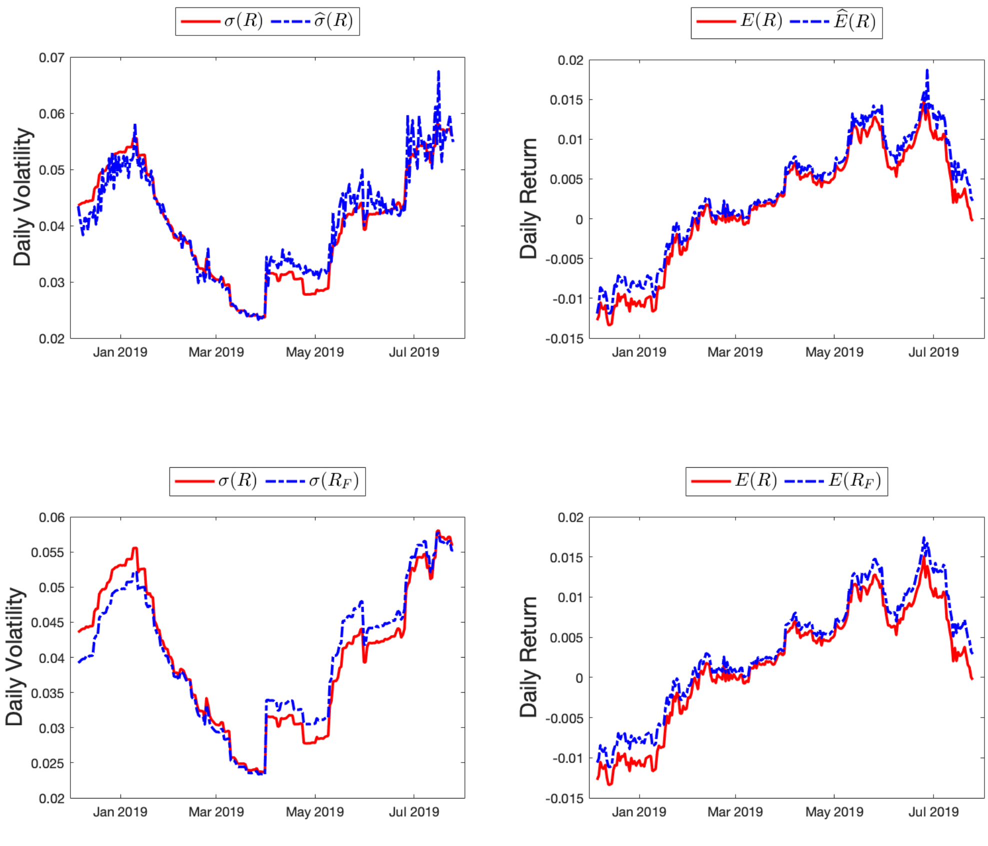

Figure 3. 60-day rolling daily mean and volatilities of returns on bitcoin futures.
Notes. Panels 1-2 report return $E(R)$ and volatility $\sigma(R)$, and their first-order approximations $\widehat{\sigma}(R)$ and $\widehat{E}(R)$, calculated using (2.14). Panels 3-4 report the comparisons between $E(R), \sigma(R)$ and $E(R_F), \sigma(R_F)$, where R_F is defined by (2.3). In all panels, the x-axis is the time window, while the y-axis is the actual value.

We calculate $\sigma(R)$ ($\sigma(R_F)$ as well) using the sample standard deviation, and $\widehat{\sigma}(R)$ using (2.14). All the volatilities are computed on a 60-day rolling window using data from the full sample. The results are plotted in Figure 3. To be specific, in Panels 1-2 of Figure 3, we draw the comparison curves of $\sigma(R)$ vs $\widehat{\sigma}(R)$ and $\mathbb{E}(R)$ vs $\widehat{\mathbb{E}}(R)$. In Panels 3-4, we plot $\sigma(R)$ against $\sigma(R_F)$, and $\mathbb{E}[R]$ against $\mathbb{E}[R_F]$ together to compare the volatilities and expectations of returns on inverse and standard futures. Recall R_F, defined by (2.3), is the return on the futures reference price F, and hence, can be seen as the return on Bitcoin standard futures. We also report the average volatilities of $\sigma(R)$ and $\sigma(R_F)$ in Table 2. The key findings are due as follows:

- The first-order approximation $\widehat{\sigma}(R)$ is as good as the commonly used sample standard deviation $\sigma(R)$. The accuracy is above 97% for $\sigma(R)$ (see Table 2). It is well known that the latter is an unbiased and consistent estimator to the true volatility.
- Volatilities $\sigma(R)$ and $\sigma(R_F)$ are very close but not identical. That means, if we use volatility (variance) as a risk measure, trading Bitcoin standard futures is as risky as trading Bitcoin inverse futures. On the average level, Table 2 confirms that Bitcoin inverse

futures are (slightly) more risky than standard futures. However, as shown by Panel 3 of Figure 3, there are also times when inverse futures are less risky than standard futures.

Table 2. 60-day rolling daily volatility.

Variable	Result
Average $\sigma(R)$	4.07%
Average $\widehat{\sigma}(R)$	4.16%
Accuracy	97.43%
Average $\sigma(R_F)$	4.03%

Notes. To calculate the standard deviations $\sigma(R^B)$, and $\sigma(R_F)$, we only need the Bitcoin futures prices. Accuracy is calculated as $|(\text{Average } \widehat{\sigma}(\cdot) - \text{Average } \sigma(\cdot))/\text{Average } \sigma(\cdot)|$.

3.3. *Volatility amplification effect and downside volatility*

One interesting finding of Proposition 2.1 is the volatility amplification effect, and here we further investigate such effect in numerical analysis. We recall that volatility amplification effect refers to the case when the inverse futures' risk $\sigma_t(R)$ is inflated above the standard futures' risk $\sigma_t(R_F)$. For standard futures, there is no amplification effect, as two risks coincide. We define the following measure for volatility amplification effect:

$$\text{Volatility Amplification Effect (VAE)} := \frac{\sigma(R) - \sigma(R_F)}{\sigma(R_F)}, \tag{3.1}$$

If $\sigma(R) < \sigma(R_F)$, we get negative VAE, which corresponds to the volatility reduction effect. Once we calculate VAE given by (3.1) on a daily basis over time, we can study the dynamic changes of volatility amplification effect during the sample period. Similar to the computations of $\sigma(R)$ and $\sigma(R_F)$ in Section 3.2, we also calculate VAE using a 60-day rolling window. The results are plotted in Figure 4. We observe that:

- When the market is in contango (ratio $S/F < 1$), the volatility $\sigma(R)$ is reduced by the ratio S/F and VAE is negative, implying that Bitcoin inverse futures are less risky than standard futures.

- When the market is in backwardation (ratio $S/F > 1$), the volatility $\sigma(R)$ is amplified by the ratio S/F and VAE is positive, implying that Bitcoin inverse futures are more risky than standard futures.

When the market is neutral ($S/F \simeq 1$), two volatilities are almost indistinguishable. These findings confirm our conjecture based on the approximation formula (2.14).

In addition, we compute the maximal and minimal amplification effect as

$$VAE_{max} = 11.09\% \qquad \text{and} \qquad VAE_{min} = -9.07\%. \tag{3.2}$$

These numbers show that the positive amplification effect is more significant than the negative reduction effect. Such an asymmetry is also found in the impact of price changes on the payoff of Bitcoin inverse futures; see Example 2.2 in [15].

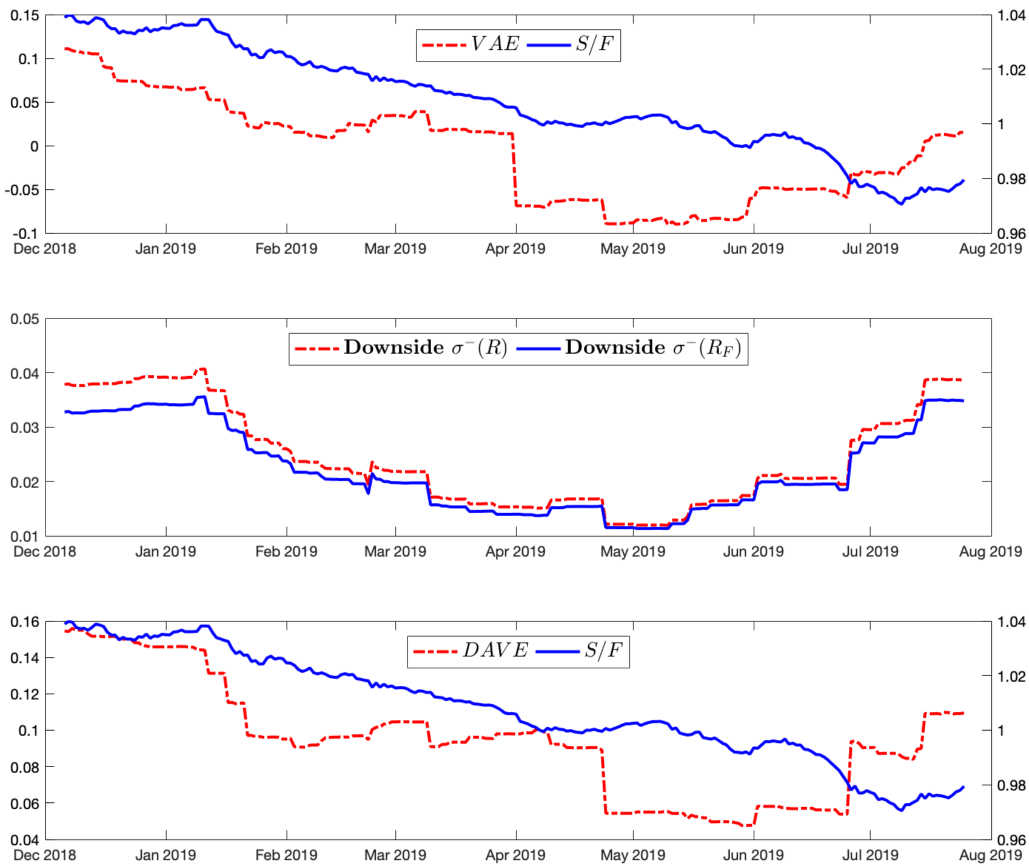

Figure 4. 60-day rolling amplification effect and downside volatility.

Notes. The right y-axis is the actual value of the ratio S/F (blue) in panels 1 and 3. The red lines in panels 1 and 3 are the amplification and downside amplification effect defined by (3.1) and (3.3). Panel 2 plots the downside volatilities, in actual value, of $\sigma^-(R)$ (red) and $\sigma^-(R_F)$ (blue). The x-axis is the time window; note that our data starts on 2018/10/7, so the first 60-day sample starts in Dec 2018.

To further investigate the volatility amplification effect induced by the non-linear payoff structure, we calculate the downside volatility (semi-deviation), which captures the downside risk of falling below the expected return. We plot the downside volatilities $\sigma^-(R)$ and $\sigma^-(R_F)$ in Panel 2 of Figure 4. It shows that Bitcoin inverse futures are more risky than standard futures, especially during market downturns. Similar to VAE in (3.1), we also define the downside volatility amplification effect (DVAE) as follows:

$$\text{Downside Volatility Amplification Effect (DVAE)} := \frac{\sigma^-(R) - \sigma^-(R_F)}{\sigma^-(R_F)}. \tag{3.3}$$

We plot the graph of DAVE in Panel 3 of Figure 4, with left y-axis for DAVE and the right y-axis for the ratio S/F. We observe positive DAVE throughout the entire period, which confirms the previous finding from Panel 2 that inverse futures bear higher risk when compared to standard futures. We then calculate the maximal and minimal downside amplification effect and obtain

$$DVAE_{max} = 15.59\% \qquad \text{and} \qquad DVAE_{min} = 4.76\%. \tag{3.4}$$

At the maximal level, Bitcoin inverse futures bear about 16% more downside risk than standard futures.

4. Conclusion

Despite with many attractive contract features, Bitcoin inverse futures are significantly different and more complex than standard futures, with nonlinear payoff function as a prominent example. In this paper, we study the mean-variance tradeoff of Bitcoin inverse futures in detail. In the analysis, we obtain explicit representations for the expectation and variance of the returns on Bitcoin inverse futures, and we further derive the corresponding first-order approximations, which are shown to be accurate estimators empirically. Based on the first-order approximations, we conduct an extensive empirical study to compare the riskiness of inverse and standard futures. Our findings show that Bitcoin inverse futures are more (resp. less) risky than standard futures when the market is backwardation (resp. contango). We further find that Bitcoin inverse futures bear higher downside risk than standard futures.

Unlike the standard futures traded on CME and CBOE, Bitcoin inverse futures are exclusively traded on exchanges that are not (or lightly) regulated by governments and authorities. In addition, our analysis and results are mainly objective and not impacted by regulations and policies. However, investors who trade Bitcoin inverse futures, depending on where they reside, may be subject to local regulations; for instance, residents in the US are not allowed to trade on many cryptocurrency exchanges, such as BitMEX and Binance (main site), and are required to report their crypto tradings in their annual tax filing. We point out several potential directions for future research. Recall that our analysis is model-free in the sense that we do not assume *a priori* a model for the spot or futures price. So, the first direction is to consider special models, such as a stochastic volatility model with correlated jumps in [19], and obtain finer results. Second, the exact representations in (2.6) and (2.8), along with their first-order approximations (2.13) and (2.14), could help construct time-series models for the Bitcoin spot price S to better fit with data (recall that Bitcoin inverse futures are the most traded product among all cryptocurrencies and their derivatives). Third, one could explore the applications of our representation results in hedging and portfolio management. As an example, the mean and variance approximations in (2.13) and (2.14) can be used to construct optimal hedging portfolios.

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Conflicts of interests

The authors declare no conflict of interest.

Authors' contribution

Jun Deng: Supervision, Formal analysis, Conceptualization, Methodology, Writing- Original draft preparation, Writing- Reviewing and Editing; **Huifeng Pan**: Conceptualization, Data curation, Software, Writing- Original draft preparation; **Shuyu Zhang**: Conceptualization, Visualization, Investigation, Software, Writing- Original draft preparation; **Bin Zou**: Conceptualization, Methodology, Writing- Reviewing and Editing.

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