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# Competitive solvers in action: strategic optimization across CoWs and multi-chain AMMs <sup>†</sup>

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## **Highlights:**

- Introduces a cross-chain intent settlement model using CoWs and multi-chain AMMs.
- Achieves 26.1%–46.1% higher completion rate versus CoWs-only execution.
- Uses an on-chain solver auction with Bayesian game theory for fair competition.
- Integrates gas, liquidity, and clearing price constraints in a unified optimization model.

**Abstract:** In the rapidly evolving decentralized finance (DeFi) ecosystem, ensuring efficient and interoperable transaction mechanisms is a critical challenge. This paper introduces a strategic optimization model for a blockchain-based token exchange platform, leveraging Coincidence of Wants (CoWs), multi-chain Automated Market Makers (AMMs), and an on-chain solver auction mechanism to enhance transaction efficiency and cross-chain interoperability in DeFi. In our model, users specify their transaction intents, while solvers, selected through a competitive auction based on game theory principles, compete to find the most efficient execution pathways, considering liquidity availability and market constraints. This approach not only facilitates seamless cross-chain transaction flows, but also optimizes the efficiency of existing solvers and reduces the reliance on centralized mechanisms. Our model's effectiveness is validated through extensive simulation experiments, where performance with various order inputs and AMM constraints demonstrates a transaction completion rate increase ranging from 26.1% to 46.1% compared to the CoWs-only model, thereby enhancing user welfare and market fairness. The proposed model offers broad applicability for efficient, interoperable cross-chain transactions, positioning it to make a significant impact on the DeFi landscape.

**Keywords:** solvers; coincidence of wants; automated market makers; user welfare optimization; cross-chain liquidity



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#### 1. Introduction

A blockchain is a ledger or database of digital transactions, shared among nodes of a computer network [1]. This computing solution has risen in popularity over the years since the first blockchain, Bitcoin, was created in 2008 [2,3]. For example, the banking and financial services sector of the blockchain market alone grew from \$1.89 billion in 2022 to \$3.07 billion in 2023 (a compound annual growth rate of 62.1%) [4]. Cryptocurrencies ("crypto") and other digital assets that operate in blockchain networks have been a significant factor in this growth, with the total crypto market capitalization doubling in 2023 [5]. Together, crypto and blockchain technology have enabled a robust market for decentralized finance (DeFi), an industry of crypto-based transactions, exchanges, and other financial services.

Within DeFi, there are a number of mechanisms that facilitate the settlement of crypto transactions. Automated market makers (AMMs) play a particularly important role. AMMs are institutions that stand ready to buy or sell assets automatically. They do so by allowing traders to place orders within the AMM using algorithmic pricing [6]. These entities are an improvement of traditional market makers, which are not automated and thus required more intensive means of establishing prices.

While AMMs are important in crypto transactions, it is possible to settle transactions without them. Notably, transactions can be settled via the Coincidence of Wants (CoWs) principle [7]: an economic phenomenon where two or more parties coincidentally hold an item or asset that the other wants. Thus, these parties can exchange directly with one another without an intermediary exchange like an AMM. In the case of crypto, a CoW occurs when transactions are coincidentally the opposite of one another (*i.e.* a transaction swapping asset A for asset B and another to swap asset B for asset A form a CoW) [8]. By removing the need for a market maker, settling a crypto transaction via a CoW requires less information to be processed on-chain. This is often more time- and cost-efficient, and thus considered preferable to AMM settlement.

Despite its many advancements such as AMMs, DeFi is still an evolving industry [9]. One persisting limitation is the lack of interoperability between blockchains. Specifically, interacting with DeFi processes across multiple blockchains is quite complex; users must navigate between many apps and wallets, perform multiple transactions, and identify opportunities quickly before they disappear. Moreover, there are many barriers between different blockchains, meaning users and subsequently liquidity cannot seamlessly transact between ecosystems. Existing crypto transaction settlement tools do not address these issues. For example, there is a lack of cross-chain AMM solutions.

As a result, in the blockchain and DeFi space, popular areas of research and development include cross-chain interoperability [10,11] and advanced intents/solvers [12,13]. However, these ideas have yet to be combined; there is not a cross-chain intent settlement framework available on the market. Such a framework would enable users to simultaneously take advantage of the benefits of both intents/solvers and cross-chain interoperability.

To this end, this paper aims to propose a strategic optimization model for enhancing solvers that compete to facilitate crypto transaction intents using CoWs and multi-chain AMMs. We further outline the expected impacts and contributions of our model to the DeFi ecosystem. When integrated with cross-chain infrastructure (like the Picasso Network [14] and Mantis [15] developed by the Composable

Foundation [16]) our optimization model simplifies cross-chain token intent settlements and token exchanges, thereby improving completion rates and delivering significant benefits to users. This paper serves as an extended version of our previous conference paper [17], providing deeper insights and expanded contributions to this topic.

The remainder of this paper is organized as follows: Section 2 defines and describes relevant concepts. Section 3 details a system designed to optimally solve intents within a cross-chain context. Section 4 further describes the key technologies of this system, including an auction-based competitive solvers model and an integrated optimization model based on CoWs and CFMMs. The experimental results are presented in Section 5. Finally, Section 6 summarizes the findings and discusses areas for future research.

## 2. Background and related work

This section outlines the foundational concepts and prior research pertinent to our study, encompassing topics such as intents and solvers, CoWs, types of market makers, and cross-chain interoperability. These areas form the theoretical underpinnings of our proposed models and are critical for understanding the subsequent discussions.

#### 2.1. Intents and solvers

In the evolving blockchain and DeFi ecosystem, "intents" and "solvers" have become pivotal concepts, exemplified by platforms like Anoma [12], SUAVE by Flashbots [18], and CoW Swap by CoW Protocol [13]. An *intent* generally refers to a user-defined set of constraints for cryptocurrency transactions, outlined as an off-chain signed message indicating desired state transitions [12]. Unlike fixed transactions, intents allow for flexibility in the execution path, aiming for optimal outcomes such as cost savings.

*Solvers*, on the other hand, are entities that devise the most efficient execution pathway for these intents. They compete to provide the best price or terms for a user's order. The successful solver executes the transaction and potentially receives rewards [19]. This competition ensures users receive favorable transaction terms. Moreover, the use of uniform clearing price batch auctions within this system helps prevent miner extractable value (MEV) attacks by eliminating disparities in order fulfillment prices, thereby safeguarding against front-running and similar strategies [8].

#### 2.2. Coincidence of Wants

*Coincidence of Wants* (CoWs) is an economic phenomenon wherein each party possess an item or items that the other party desires, and thus are able to exchange these items directly to meet their wants [8]. When applying this concept to blockchain and cryptocurrency, a CoW enables two users' orders to be matched for settlement without the need for an external market maker or liquidity provider. In the case of intents, this principle means that a user's intent can coincidentally be the opposite of another user's intent (e.g. one intent to swap A for B and another to swap B for A form a CoW).

In the intent settlement framework discussed presently, CoWs are one means in which solvers may settle user intents [19]. This is arguably the optimal means for intent settlement whenever CoWs are available, due to the lack of a need for an external market or liquidity (which thus eliminates any fees associated with use of external markets). Therefore, in the optimization model presented in this paper,

CoWs are a prioritized form of intent settlement.

## 2.3. AMMs and CFMMs

Automated market makers automate the traditional market-making process, which typically requires maintaining a constant presence to buy or sell assets. By enabling users to place orders at algorithmically determined prices, AMMs facilitate efficient trading in decentralized markets [6]. By contrast, *constant function market makers* (CFMMs) are a specialized subset of AMMs that operate through smart contracts. Liquidity providers contribute capital to CFMMs, which then continuously facilitate trades based on a predefined trading strategy articulated through a function of its asset reserves [20]. This model ensures liquidity and price stability within DeFi platforms.

Notable types of CFMMs include the constant product model, exemplified by Uniswap v2 [21], which maintains a constant product of the reserves of two assets to ensure liquidity regardless of market size. The constant sum model [6] trades assets such that the sum of the reserves remains unchanged. This model is suitable for markets with stable asset values. Meanwhile, the constant mean model, used by Balancer [22], employs a weighted average of several assets to define the trading function, accommodating diversified liquidity pools.

#### 2.4. Cross-chain interoperability

A pivotal concept to this research is cross-blockchain interoperability, which encompasses a range of terminologies such as cross-blockchain communication, cross-domain interoperability, and cross-chain operations. *Cross-chain interoperability* can be succinctly defined as "the process of asset interoperability between two relatively independent blockchain ledgers" [10]. This involves not only the transfer of assets but also the synchronization of information between a source chain, where a transaction originates, and a target blockchain, where the transaction is executed, typically facilitated by some cross-chain messaging protocol [23].

The significance of cross-chain interoperability lies in its ability to enable the seamless flow of messages and assets across independent blockchain ecosystems, thereby unlocking new use cases within the cryptocurrency domain. Such capabilities include efficient cross-chain asset bridging, streamlined governance across protocols deployed on multiple chains, and facilitation of cross-chain DeFi activities. These activities include liquidity provisioning, lending and borrowing, and yield aggregation, all of which offer substantial opportunities for users to earn returns [24]. In light of these opportunities, a variety of cross-chain interoperability protocols have been developed, garnering considerable attention in academic and practical blockchain research. Notably, the Inter-Blockchain Communication (IBC) Protocol stands out as a prime example of such advancements [11].

Despite the advancements and evident benefits of cross-chain interoperability, the market still lacks a comprehensive cross-chain intent settlement framework. This paper proposes an optimization model for solvers that is instrumental in establishing a robust cross-chain intent settlement framework, thus addressing this significant gap in the field.

## 3. System overview

The system presented in this section is an intent settlement framework designed to optimize the settlement of cryptocurrency transactions across multiple blockchain networks. The system's goal is to enhance the DeFi ecosystem's efficiency and interoperability.



Figure 1. System overview of the proposed intent settlement framework.

The system overview is depicted in figure 1. The process begins when users submit their intents. The solver protocol then collects these intents. Each intent specifies parameters of the desired transaction, such as swaps between different cryptocurrencies. At the core of the model is a solver, which executes an optimization algorithm to match orders. In practice, distributed solvers compete for the right to solve optimized models. The solver that provides the most user welfare will be selected as the winner and granted the right to settle the intent.

In this system, solvers make an initial attempt to ensure that direct matches between opposite user intentions are prioritized using the principle of CoWs, thus reducing transaction costs and increasing speed. This matching is facilitated through a uniform price batch auction system where transactions are aggregated and executed at a uniform price. This makes transaction order irrelevant within the block, undermining the ability for MEV bots to extract value [8]. For intents that cannot be directly matched via CoWs, the solver routes these orders to CFMMs located on various blockchain networks. This ensures that liquidity is utilized efficiently and that transactions are completed even when direct counterparties are not immediately available. The architecture supports multiple CFMMs across different chains, reflecting a robust approach to handling a diverse set of transaction scenarios and user demands. The model's integration of CoWs with cross-chain CFMM routing optimizes the flow of digital assets across disparate blockchain environments. Accordingly, the model significantly enhances user experience by providing faster, cheaper, and more reliable transaction settlement.

## 4. Key techniques

## 4.1. Auction-based competitive solvers

In our model, multiple solvers compete to execute tasks or optimize transactions based on specific criteria set by users. To effectively select the most suitable solver from a pool of *n* potential candidates, we propose using a first-price reverse bid auction mechanism. In this setting, solvers submit bids indicating the lowest price at which they are willing to perform the task, with the goal of being selected while maximizing their profit. This auction model incentivizes solvers to balance between offering competitive bids and maintaining profitability, ensuring that the user receives the best possible service at the lowest cost.

To model the competitive bidding process among solvers, we adopt a first-price reverse bid auction mechanism, where each solver submits a bid based on their internal cost of performing the task and their expected utility. Solvers do not have complete information about the bids of other solvers, thus creating an environment of incomplete information. This leads us to frame the auction as a Bayesian game where solvers only know their own costs and have beliefs about the distribution of other solvers' costs.

## 4.1.1. Assumptions

- There are *n* solvers competing in the auction, and each solver *i* submits a bid  $b_i$ , which is a function of their internal expected cost for executing the task, denoted as  $v_i$ .
- $v_i$  is private information to solver *i* and represents the true cost or value that solver *i* assigns to the task. It is assumed to be independently and identically distributed (i.i.d.) over the interval  $[v^{\min}, v^{\max}]$  with cumulative density function F(v) and probability density function f(v).
- The objective for each solver is to submit the lowest bid while ensuring they maximize their utility and win the auction by offering a competitive price lower than other n 1 solvers.

## 4.1.2. Utilities

In a first-price reverse bid auction, the solver with the lowest bid wins the auction and is selected to execute the task. The selected solver's utility is the difference between their bid and their internal cost of performing the task, minus any associated blockchain transaction fees  $C_P$ . The utility function for solver *i* is defined as:

$$u_{i}(b_{i}, v_{i}) = \begin{cases} b_{i} - v_{i} - C_{P} & \text{if } b_{i} < b_{j} \text{ for all } j \neq i \\ -C_{P} & \text{otherwise} \end{cases}$$

#### 4.1.3. Bayesian Nash Equilibrium

Let the strategy of each solver be defined by a bidding function  $b_i(v_i)$ , which maps the solver's private cost  $v_i$  to a bid. A Bayesian Nash Equilibrium (BNE) is achieved when each solver's bidding strategy maximizes their expected utility, given their beliefs about the other solvers' bids. The equilibrium bidding strategy is given by:

$$b_{i}^{*}(v_{i}) = v_{i} + \frac{\int_{v_{i}}^{v^{max}} (1 - F(v_{i}))^{n-1} dv}{(1 - F(v_{i}))^{n-1}}$$

This equation indicates that each solver's optimal bid is a function of their private cost  $v_i$  and the distribution of the other solvers' costs. The equilibrium bid balances the need to submit a competitive bid with the desire to maximize profit.

*Proof.* To derive the optimal bidding strategy for solver *i*, we consider the expected utility of solver *i* if they bid  $b_i$  while the others bid according to their equilibrium strategies. Solver *i* will win the auction if their bid is lower than all the other bids, *i.e.*,

$$\Pr(b_i < b_j \text{ for all } j \neq i) = \Pr(v_j > b_j^{-1}(b_i), \forall j \neq i).$$

The probability of winning is therefore:

$$\Pr\left(\sum_{j=1}^{n-1} \mathbb{1}(v_j > b_i^{-1}(b_i)) \ge n-1\right) = \left[1 - F(b^{-1}(b_i))\right]^{n-1}$$

The expected utility for solver *i* is:

$$\mathbb{E}[u_{i}(b_{i},v_{i})] = (b_{i}-v_{i}-C_{P}) \cdot [1-F(b^{-1}(b_{i}))]^{n-1}$$

Taking the first derivative with respect to  $b_i$  and setting it to zero gives the first-order condition for maximizing the utility:

$$\frac{\partial}{\partial b_{i}}\left[\left(b_{i}-v_{i}-C_{P}\right)\cdot\left(1-F\left(b^{-1}\left(b_{i}\right)\right)\right)^{n-1}\right]=0.$$

By solving this differential equation, we obtain the optimal bidding function:

$$b_{i}^{*}(v_{i}) = v_{i} + \frac{\int_{v_{i}}^{v^{max}} (1 - F(v_{i}))^{n-1} dv}{(1 - F(v_{i}))^{n-1}}.$$

Based on the auction model discussed above, figure 2 presents an overview of the competitive solver model using an on-chain auction mechanism. The process begins with the Solver Protocol deploying an auction smart contract on the blockchain (Step 1). Once the contract is deployed, the Solver Protocol gathers transaction orders and submits them to the smart contract (Step 2). Candidate solvers, each equipped with distinct capabilities and cost functions, compute optimal bids based on their Bayesian Nash Equilibrium (BNE) strategy and submit hashed bids to the contract to ensure privacy (Step 3). In the next phase, an on-chain first-price sealed-bid (FPSB) auction is conducted (Step 4) to select the solver with the lowest bid, thus identifying the Winner Solver. The winning solver then contributes its computing power to perform optimization, processing the orders against a uniform clearing price (UCP) batch auction within the optimization model (Step 5). Finally, the winning solver is compensated through an on-chain cryptocurrency payment (Step 6), rewarding it for efficiently completing the task. A descriptive algorithm about this flow is also shown in Algorithm 1.

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**Require:**  $N_s$  (total number of solvers),  $v_i$  (private cost of solver *i*), F(v) (cumulative distribution function of costs), f(v) (probability density function of costs),  $C_P$  (blockchain transaction fee)

Ensure: Selected solver with lowest bid for task execution

- 1: Initialize hash list H = []
- 2: Initialize bid list B = []
- 3: **for** i = 1 to  $N_s$  **do**
- 4: Solver *i* observes its private cost  $v_i$
- Compute optimal bid  $b_i$  using the Bayesian Nash Equilibrium formula: 5:

$$b_i = v_i + \frac{\int_{v_i}^{v_{max}} (1 - F(v_i))^{N_s - 1} dv}{(1 - F(v_i))^{N_s - 1}}$$

Compute utility of solver *i*: 6:

$$U_i = b_i - v_i - C_P$$

- if  $U_i < 0$  then 7:
- Solver *i* does not participate in the auction 8:
- 9: else
- 10: Compute hash of bid  $b_i$ :  $h_i = hash(b_i)$
- Submit hash  $h_i$  to hash list H on the blockchain 11:
- 12: end if
- 13: end for
- 14: Wait until all solvers have submitted their hashes in H
- 15: **for** i = 1 to  $N_s$  **do**
- 16: Solver *i* submits their actual bid  $b_i$
- 17: Verify that  $hash(b_i) = h_i$
- 18: Submit bid  $b_i$  to bid list *B* on the blockchain
- 19: end for
- 20: Sort bid list B in ascending order
- 21: Select solver with the lowest bid, solver  $i^*$ , where:

 $i^* = \arg \min B$ 

22: return Selected solver  $i^*$  and its bid  $b_{i^*}$ 

▷ Empty list to store hashed bids

▷ Empty list to store revealed bids

▷ Each solver calculates its bid and utility

▷ Phase 2: Each solver reveals their bid

▷ If utility is negative, solver rejects the task

> Check if revealed bid matches submitted hash

Article



Figure 2. Competitive solver model based on on-chain auctions.

## 4.2. Main optimization model for CoWs

In this section, we discuss the CoWs optimization model that dictates the decision-making process among users. This is designed to maximize trading utility. The utility function integrates the individual utilities of a set of trading orders, factoring in the specifics of each order type—either limit-buy or limit-sell—and relevant parameters.

The utility functions for limit-buy and limit-sell orders are designed to accommodate both full and partial executions at a specified limit exchange rate  $\pi$ . This ensures adherence to quantity and price constraints. For a limit-buy order involving the purchase of *x* units of token *k* and the sale of *y* units of token *j* at rate  $\pi$ , the utility is  $(x \cdot \pi - y) \cdot p_{b,k}$ . Here,  $x \cdot \pi$  is the ideal payment in token *j* for *x* units of token *k*, and *y* is the actual payment. The utility captures the net benefit, factoring in the internal clearing price  $p_{b,k}$ . Similarly, the utility for a limit-sell order which entails selling *y* units of token *k* to buy *x* units of token *j* at rate  $\pi$ , is calculated as  $(x - \frac{y}{\pi}) \cdot p_{b,j}$ . This formula evaluates the trade's efficiency when fully or partially executed.  $\frac{y}{\pi}$  represents the received equivalent in token *j*. The difference between the ideal and actual tokens received, multiplied by the internal price  $p_{b,j}$ , quantifies the utility derived from the transaction. Thus, we can define objective as:

#### 4.2.1. Objective

The main goal of the CoWs optimization model is to maximize the total trading utility  $u_{CoWs}$ :

$$\max\left[u_{CoWs} = \sum_{i=0}^{N_o - 1} (u_i - g_i)\right]$$

where  $u_i$ ,  $g_i$ , and  $N_o$  are defined as follows:

- If order i is a limit-buy order, then  $u_i = (x_i \cdot \pi y_i) \cdot p_{b,k}$ .
- If order i is a limit-sell order, then  $u_i = (x_i \frac{y_i}{\pi}) \cdot p_{b,j}$ .

•  $N_o$  is the total order number.

## 4.2.2. Constraints

The trading rules for CoWs are adapted from the CoW Swap model [13]. CoW Swap uses the following definition of a set of valid trading constraints for limit-buy and limit-sell orders based on exchange rates and order size limits: "the order either is fully executed and the limit price is respected, or it is partially executed and is traded at its limit price [25]". To simulate this scenario, we can set a binary variable  $b_i$  to enforce these constraints.  $b_i = 1$  corresponds to one set of conditions and  $b_i = 0$  corresponds to another. **Trading volume limit:** We first set the maximum values of  $x_i$  and  $y_i$ . For all  $i \in N_0$ , we set:

$$x_i \le x_{max}$$
$$y_i \le y_{max}$$

**Trading rules for limit-buy orders:** For all  $i \in N_0$  with limit-buy order, we set  $M = x_{max} + 0.001$ , where 0.001 serves as an arbitrary small adjustment value to ensure the constraints are properly implemented.

$$x_i \cdot (\pi - 0.001) \cdot b_i \le y_i \le x_i \cdot \pi$$
$$x_i + 0.001 - M \cdot (1 - b_i) \le x_{\max} \le x_i + M \cdot b_i$$

**Trading rules for limit-sell orders:** Similarly, for all  $i \in N_0$  with limit-sell order, we set  $N = y_{max} + 0.001$ :

$$x_i \cdot (\pi - 0.001) \cdot b_i \le y_i \le x_i \cdot \pi$$
$$y_i + 0.001 - N \cdot (1 - b_i) \le y_{\max} \le y_i + N \cdot b_i$$

Here, we employ the Big-M formula, using large constants M and N for limit-buy and limit-sell orders respectively. This introduces a linear relaxation of binary constraints, simplifying the solution process. For both order types, setting  $b_i = 1$  ensures that orders are executed exactly at the specified limit exchange rate  $(\frac{y_i}{x_i} = \pi)$ , with  $x_i < x_{\text{max}}$  for buys and  $y_i < y_{\text{max}}$  for sells, respectively. Conversely, when  $b_i = 0$ , the constraint  $\frac{y_i}{x_i} \leq \pi$  maintains that no orders exceed the limit exchange rate, with  $x_i$  set at  $x_{\text{max}}$  for buys and  $y_i$  at  $y_{\text{max}}$  for sells.

These constraints effectively capture the dynamics of trading with limit orders, accurately reflecting the strategic decision-making based on traders' risk tolerance and market expectations. Resultantly, we enhance the realism and applicability of our trading model.

#### 4.3. Combined volume constraints

To execute asset swaps, we initially use the CoWs method due to its efficiency in facilitating direct trades among users. If a trade cannot be fully executed using CoWs, we employ CFMMs to process the remaining portion. This approach ensures swift and streamlined execution, optimizing the trading process and minimizing potential delays and slippage. In cases where CoWs is insufficient, CFMMs serve as an alternative to complete the transaction.

#### 4.3.1. Partial execution with CoWs

Given the available volume limit for the CoWs model:

$$x_i \le V_{\text{CoWs}, x_i}$$
$$y_i \le V_{\text{CoWs}, y_i}$$

Here,  $V_{\text{CoWs},x_i}$  and  $V_{\text{CoWs},y_i}$  are maximum volumes set by the solver, adjusted dynamically based on operational needs and strategy to optimize trading efficiency. Conversely,  $x_{\text{max}}$  and  $y_{\text{max}}$  (which are discussed in the previous section) are user-set upper limits on the order sizes, defining the maximum tokens to be traded. The distinction is critical as  $V_{\text{CoWs},x_i}$  can be altered to facilitate more transactions within the limits of  $x_{\text{max}}$  and  $y_{\text{max}}$ , or to manage risk and adapt to market conditions. This ensures transactions under CoWs operate within user constraints and maintain flexibility in execution.

#### 4.3.2. Remaining trades in CFMMs

When a trade surpasses the available volume in CoWs, the residual trade for assets is represented by  $\Delta x_i$  and  $\Delta y_i$  and is redirected to the CFMMs:

$$\Delta x_i = \max(0, x_i - V_{\text{CoWs}, x_i})$$
$$\Delta y_i = \max(0, y_i - V_{\text{CoWs}, y_i})$$

Here, the max function ensures the residual trade amounts ( $\Delta x_i$  and  $\Delta y_i$ ) are not less than zero. Specifically, if the volume of the trade exceeds the available volume in CoWs orders, the surplus is calculated using this function and redirected to CFMMs. This ensures that any excess demand is properly managed.

CFMMs, with their inherent constant functions, are equipped to handle these trades. Taking the constant product CFMM as an example, for a liquidity pool with assets  $e_{1,i}$  and  $e_{2,i}$ , the constant product is:

$$k_i = e_{1,i} \times e_{2,i}$$

For a given trade volume  $\Delta x_i$  in asset 1, the corresponding trade in asset 2 is:

$$\Delta y_i = \frac{k_i}{e_{1,i} + \Delta x_i} - e_{2,i}$$

To maintain pool liquidity, we can set:

$$\Delta x_i \leq \alpha \times e_{1,i}$$
$$\Delta y_i \leq \alpha \times e_{2,i}$$

where  $\alpha$  is the pool liquidity coefficient.

#### 4.4. Nested optimization model for CFMMs

To optimize a trade in CFMMs, we seek to maximize a utility function,  $U(\Psi)$ , that accounts for the total value of the trade and subtracts any fixed transaction costs. The variables and constraints encapsulate the trade volumes, liquidity considerations, and fees associated with the CFMMs.

To model an optimization problem for CFMM-based optimal routing across multiple blockchains, we need to incorporate an additional dimension that represents each blockchain platform. Let us denote the set of blockchain platforms as B and its cardinality (the number of blockchains) as n.

#### 4.4.1. Objective

The main goal is to maximize the utility derived from trading while accounting for fixed transaction costs that may vary across different blockchains.

$$\max\left[u_{CFMMs} = U(\Psi) - \sum_{j=0}^{n-1} g'_j \eta_j\right]$$

where  $U(\Psi)$ ,  $\Psi$ ,  $g'_{j}$ , and  $\eta_{i,j}$  are defined as follows:

- $U(\Psi)$  is an equation mapping the number of traded tokens to utility, which can be defined as an arbitrary function.
- $\Psi$  is the total net number of tokens tendered to the CFMMs in the network. Specifically:

$$\Psi = \sum_{j=1}^{n} \sum_{i=1}^{m} A_{i,j} (\Delta y_{i,j} - \Delta x_{i,j})$$

where  $A_{i,j}$  is the coefficient matrix that shows the weights of trade on CFMM *i* on blockchain *j*.

- $g'_i$  is the constant cost incurred by the order execution.
- $\eta_{i,j}$  is a binary decision variable indicating whether or not to trade on CFMM *i* on blockchain *j*.

## 4.4.2. Constraints

The constraints considered for multi-chain CFMM transactions include:

**Liquidity Constraint:** This ensures the liquidity of any CFMM does not go below its initial state after a trade.

$$\varphi_{i,j}(R_{i,j} + \gamma_{i,j}\Delta x_{i,j} - \Delta y_{i,j}) \ge \varphi_{i,j}(R_{i,j}) \quad \forall i, j$$

where  $\varphi_{i,j}$  is the trading function of CFMM *i* on blockchain *j*. Examples of trading functions are the product function, the sum function, and the weighted geometric mean function [26].  $R_{i,j}$  is the current reserves and  $\gamma_{i,j}$  is the commission fee.

**Trading Volume Limit:** This bounds the trade volume based on the liquidity and decision to trade on a specific CFMM on a blockchain.

$$0 \le \Delta x_{i,j} \le \eta_{i,j} \Delta_{x,i,j}^{\max} \quad \forall i, j$$
  
 $\Delta y_{i,j} \ge 0 \quad \forall i, j$ 

## 4.5. Other considerations

## 4.5.1. Uniform clearing prices

For each batch (for CoWs and CFMMs), a uniform price is applied to constrain market volatility. Uniform pricing ensures all transactions in a batch are executed at the same price, enhancing market fairness and stability.

- $\forall i \in \{0, 1, \dots, N_o 1\}, x_i \cdot p_{b,j} = y_i \cdot p_{b,k}$
- $\forall i \in \{0, 1, \dots, N_c 1\}, a_{1,i} \cdot p_{b,j_{u,i}} = a_{2,i} \cdot p_{b,k_{u,i}}$

## 4.5.2. Gas cost incorporation

In the optimization of trading models, accounting for gas costs is critical for achieving realistic assessments of net gains and losses. Each order *i* incurs not only a straightforward gas cost  $g_i$  in the CoWs model, but also complex and varied costs when routed through CFMMs. Specifically, CFMM gas costs  $g'_i$  can be broken down into three main components:  $g_{swap,i}$ ,  $g_{delay,i}$ , and  $g_{network,i}$ .

$$g'_i = g_{\text{swap},i} + g_{\text{delay},i} + g_{\text{network},i}$$

 $g_{swap,i}$  covers transaction fees for asset swaps facilitated by the market maker's smart contracts.  $g_{delay,i}$  accounts for costs associated with execution delays, which can arise from blockchain congestion or sequential processing, reflecting price slippage and opportunity costs. Lastly,  $g_{network,i}$  includes fees paid to the blockchain network, varying with network demand, which ensure prompt trade execution. These components together encapsulate the primary expenses incurred during CFMM routing.

Algorithm 2 Integrated Optimization Algorithm for Solvers

**Require:**  $N_0$  (total number of orders),  $N_c$  (total number of CFMMs), **x** and **y** (trade volume vectors),  $\pi$  (exchange rate),  $p_{b,k}$  and  $p_{b,j}$  (buy and sell clearing prices),  $g_i$  (gas costs for CoWs),  $g'_i$  (gas costs for CFMMs)

**Ensure:** Optimized trade execution strategy via CoWs and CFMMs

1: Define utility functions  $u_i$  for orders and a global utility maximization goal

```
2: for i = 0 to N_0 - 1 do
                                                                                              ▷ Iterate through each order
        if Order i is limit-buy then
 3:
 4:
            Compute utility u_i = (x_i \cdot \pi - y_i) \cdot p_{b,k}
        else if Order i is limit-sell then
 5:
            Compute utility u_i = (x_i - \frac{y_i}{\pi}) \cdot p_{b,i}
 6:
 7:
        end if
 8:
        Apply CoWs trading rules to attempt full execution
9:
        if order i not fully executed then
10:
            Calculate unexecuted volumes \Delta x_i, \Delta y_i
            Route remaining volume to CFMMs for execution
11:
        end if
12:
13: end for
14: for i = 0 to N_c - 1 do
                                                                                  ▷ Optimize trade execution in CFMMs
        Apply CFMM trading constraints for liquidity and execution
15:
16: end for
17: Compute total gas costs g_{total,i} = g_i + g'_i for each order
18: Apply gas cost constraints and ensure net utility exceeds g_{total,i}
19: return Comprehensive trade execution strategy
```

The total gas cost for order *i* is then:  $g_{total,i} = g_i + g'_i$ . The constraints related to the total gas costs include:

$$\sum_{i=1}^{N_o} g_{total,i} \le G$$

Based on the whole model presented in this section, the flow of transaction processing can be described as an integrated optimization algorithm (as shown in Algorithm 2).

## 5. Experimental results



**Figure 3.** Results of different simulation experiments with various cost distributions for solver auction optimization, showing different configurations for solver number (n) and transaction fee multiplier  $(c_p)$ .

This section shows the experimental results of our proposed model. We first present the evaluation of the solver auction model, which includes the simulation of the real-world solver auctions and the evaluation of the smart contract. Then, we evaluate our optimization model. This includes the experimental setup as well as the experimental results for the CoW, CFMM and integrated optimization models.

## 5.1. Solver auction model evaluation

Figure 3 presents the results from various simulation experiments investigating the effects of different cost distributions on solver auction model (introduced in Section 4.1.). The evaluation metrics include Social Welfare Distribution, Winning Price Distribution, Winning Utility Distribution, and Winning Frequency Distribution across multiple configurations of solver numbers (n) and transaction fee multipliers ( $c_n$ ).

## 5.1.1. Social welfare distribution

Across different configurations, social welfare varies with the cost distribution type (uniform, normal, or exponential). For configurations with n = 10 solvers, social welfare tends to be higher for uniform and exponential distributions. When the number of solvers increases to n = 20, social welfare generally decreases across all cost distributions, and the average social welfare of the three distributions converges to the same. This suggests that more solvers may reduce auction social welfare, which may be due to increased competition or the transaction costs that each solver needs to pay.

## 5.1.2. Winning price distribution

The winning price for auctions generally appears highest under the normal cost distribution for both n = 10 and n = 20 configurations. In contrast, winning prices are notably lower for exponential cost distributions, suggesting that exponential distributions may lead to more aggressive bidding, resulting in lower prices. As the number of solvers increases (e.g., n = 20), winning prices become more competitive, likely due to the heightened competition driving prices down. Moreover, it was observed that variations in  $c_p$  have an insignificant impact on the winning price. This is likely because  $c_p$ , as a fixed cost, is consistent across all solvers and therefore does not introduce significant differences in their bidding strategies.

#### 5.1.3. Winning utility distribution

Solver utility, reflecting the benefit to the winning solver, is also influenced by the cost distribution and transaction fee multiplier. With a lower transaction fee multiplier ( $c_p = 1$ ), utility is slightly higher under normal and uniform cost distributions. However, as transaction fees and solver numbers increase ( $c_p = 2$ , n = 20), winning utility under uniform and exponential distributions becomes almost zero, indicating that higher transaction fees and solver numbers reduce solver utility in the system, potentially discouraging solver participation.

#### 5.1.4. Winning frequency distribution

The bar charts display the frequency with which each solver wins under different cost distributions. For uniform and normal distributions, the winning frequencies of different solvers are similar and alternate consistently, showing no clear dominance. In contrast, under the exponential distribution, a few solvers tend to dominate, winning significantly more frequently. This dominance becomes even more pronounced as the number of solvers increases to n = 20, suggesting that the exponential distribution amplifies competitive advantages among certain solvers in larger solver pools.

The above simulations highlight the importance of carefully considering fee structures and cost distributions to optimize auction-based solver systems, as they directly influence both system-wide welfare

and individual solver incentives.

To enable on-chain execution of the solver auction, we chose Ethereum as the platform for developing the smart contract. The interface, shown in figure 4, along with the analysis in figure 5, provides key insights into gas consumption and costs associated with the auction process, particularly in relation to the number of participants. As illustrated in figure 4, the most significant gas consumption occurs during the contract deployment and order submission phases, which are essential steps in the auction. These two actions account for the bulk of gas usage, highlighting them as priority areas for optimization to achieve meaningful cost reductions. By contrast, other actions, such as bid revelation and payment distribution, use significantly less gas, suggesting a more efficient allocation of resources for these processes. Meanwhile, figure 5 explores the relationship between the number of bidders and gas costs. The data reveals that as bidder numbers increase, both protocol gas rises steadily, while solver total gas consumption grows more sharply, indicating greater sensitivity to bidder count. Interestingly, the average gas cost per solver remains relatively stable and even shows a slight decline as bidder numbers rise. This reduction can likely be attributed to fixed costs, such as protocol overhead, being distributed across a larger pool of solvers, thereby lowering the per-solver average.



Figure 4. Gas usage and costs per action in solver auction contract.



Figure 5. Gas cost analysis for different numbers of bidders.

## 5.2. Integrated optimization model evaluation

## 5.2.1. Experiment settings

**Solver Configuration.** The experiments were conducted using the Gurobi Optimizer version 9.5.1. The optimization model is implemented using Python.

**Order Details.** Orders are represented as tuples consisting of buy/sell token indices *j* and *k*, buy and sell limits  $x_m$  and  $y_m$ , exchange rate  $\pi$ , and order type *t* (where 'b' indicates a limit-buy order and 's' indicates a limit-sell order). Below are the details of the simulated five orders, as shown in Table 1.

Order	j	k	$x_m$	Ут	$\pi$	t
1	0	1	11	74	6.73	b
2	0	1	65	10	0.15	b
3	0	1	57	89	1.56	b
4	1	0	73	100	1.37	b
5	1	0	35	56	1.6	8

**CFMM Configuration.** Experiments involved five simulated CFMMs. Each was on a different blockchain with fixed transaction costs, fees, token indices, and specific reserves, as summarized in Table 2.

CFMM	Transaction Cost	Fee	<b>Token Indices</b>	Reserves
1	0.1	2%	[0, 1, 2]	[3, 0.2, 1]
2	0.2	1%	[0, 1]	[10, 1]
3	0.15	4%	[1, 2]	[1, 10]
4	0.25	3%	[0, 2]	[20, 50]
5	0.1	1%	[0, 2]	[10, 10]

Table 2. CFMM transaction costs, fees, reserves, and token indices.

## 5.2.2. Optimization model of CoWs

The four figures (Section 5.2.2.) in the first row of figure 6 depict variations in objective values in relation to order buy and sell limits, under different exchange rate scenarios. Notably, the first figure illustrates a declining trend in objective values as the buy limit increases for a limit-buy order scenario. This suggests that higher buy limits may lead to less favorable outcomes under certain conditions, potentially due to diminishing returns on increased buy limits. Conversely, the second figure in the row shows an ascending trend in objective values with increasing sell limits, indicating that higher sell limits can enhance outcomes by facilitating better matching opportunities or more favorable sell conditions. The third and fourth diagrams in the sequence further investigate the complex interplay between order limits and objective values. Within the context of a limit-sell order, the objective value is shown to escalate swiftly. This value reaches a peak as both the buy limit and sell limit are increased, then declines and subsequently stabilizes. The time required to reach this peak correlates with the value of  $\pi$ . These observations underscore the intricacies involved in optimizing trading strategies within a simulated CoWs environment.

The second row of figure 6 presents figures that explore the relationship between clearing prices and various factors such as order limits and pricing strategies. The plots illustrate how clearing prices adjust in response to changes in the order buy and sell limits across different settings. In scenarios involving both limit-buy and limit-sell orders, the variation in the clearing price exhibits greater stability upon incrementing the sell limit. Here, the clearing price of token 1 consistently remains marginally higher, albeit with minimal fluctuation. Conversely, when the buy limit is elevated, the clearing prices of the two tokens exhibit more pronounced shifts and may even intersect. These dynamics may be attributed to the intricacies inherent in the settings of the order parameters and the constraints of the model.

The final row of figure 6 provides a detailed examination of the number of tokens transacted, exploring how this metric adapts to varying buy and sell limits as well as exchange rates. The analysis reveals that for limit-buy orders, as the buy limit increases, the number of tokens initially rises to a peak and then sharply declines. This trend suggests that, within a certain range of increased buy limits, initial increments can enhance economic benefits such as average pricing. However, surpassing a specific threshold may lead to increased costs due to constraints imposed by other market conditions. For limit-sell orders, the number of tokens remains more stable with increasing sell limits, indicating that the model is less sensitive to changes in sell limits.

## **Blockchain**

## **Article**



**Figure 6.** Results of different simulation experiments for the optimization model of CoWs (fully executed).

## 5.2.3. Optimization model of CFMMs

The diagrams in figure 7 offer a comprehensive analysis of dynamics across three different CFMM types: (1) constant product (using Uniswap v2 [21] as an example); (2) constant sum [6]; and (3) constant mean (using Balancer [22] as an example). Each set of four figures within this section explores the impact of varying parameters—blockchain transaction costs, trade volume limitations, CFMM commission fees, and reserves—on the objective value in relation to the number of token 1. Below are analyses for each CFMM type based on the figures:

#### **Blockchain**

#### **Article**



**Figure 7.** Results of different simulation experiments for the optimization model of CFMMs routing (fully executed).

The first set of figures (the first row in figure 7) illustrates the behavior of Uniswap v2 under different market conditions. Notably, the impact of blockchain transaction costs on the objective value shows a pronounced decrease as costs increase. This reflects the sensitivity of Uniswap v2 to on-chain transaction fees. Trade volume limitations reveal that tighter constraints lead to diminished objective values, indicating the critical role of liquidity in optimal market functioning. Similarly, variations in CFMM commission fees highlight the trade-offs between transaction costs for users and revenue for liquidity providers. Here, higher fees leading to lower objective values. Lastly, increasing the reserves demonstrates a positive effect on the objective value, emphasizing the importance of ample liquidity in fostering a robust trading environment.

For Constant Sum and Balancer CFMMs, the change in objective value follows a similar trend. Comparatively, the growth trends of the three CFMMs vary significantly with increases in the number of token 1. Both Uniswap v2 and Balancer follow a logarithmic growth in objective value, whereas Constant Sum CFMMs exhibit initial linear growth before stabilizing. Overall, under similar data settings, Balancer achieves the highest objective values, followed by Uniswap v2, with Constant Sum CFMMs trailing. This comparison underscores the distinct operational dynamics and efficiency of each CFMM, highlighting Balancer's superior ability to optimize trading outcomes through strategic liquidity management.

#### 5.2.4. Integrated optimization model

In figure 8, we analyze and compare the completion rates of CoWs and CFMMs for executing trades under various order constraints. We include both buy and sell orders with limit-buy and limit-sell conditions. Through a series of graphs, it becomes evident that the completion rate of CoWs and CFMMs varies significantly depending on the order type and limit. In general, the percentage of total tokens executed through CoWs surpasses that of CFMMs, indicating a higher efficiency in certain market conditions. Conversely, CFMMs have a higher execution ratio in the case of limit-sell orders. This is due to their flexible liquidity pools, which more effectively match sell orders with buy orders. Moreover, from a holistic perspective, the results show that our integrated optimization model results in a significant improvement in the order completion rate (26.1% to 46.1%) compared to the CoWs-only model under different experimental settings. Thus, our model improves user welfare and market fairness significantly.



**Figure 8.** Results of different simulation experiments with CoWs and CFMM routing integration optimization models.

## 6. Conclusion and future work

In this paper, we presented an intent settlement framework and a novel strategic optimization model for enhancing solvers that facilitate cryptocurrency transaction intents using the principles of CoWs and multi-chain AMMs. This model aims to overcome interoperability and efficiency challenges in DeFi ecosystems. By integrating cross-chain capabilities, our framework enables seamless interactions across different blockchain platforms, thereby enhancing liquidity and transaction flexibility. Furthermore, we introduced an on-chain solver auction model based on Bayesian game theory, allowing solvers to compete in a decentralized and transparent manner. This auction mechanism ensures fair selection of solvers while optimizing costs and improving system efficiency. Our experimental results demonstrate that the proposed model effectively optimizes solver operations, ensuring transactions are settled efficiently and at minimal costs. By leveraging multi-chain AMMs and the CoWs principle, the model reduces the dependency on centralized mechanisms and enhances the privacy and security of transactions. The implementation of this model, combined with cross-chain bridging infrastructure developed by Composable, validates our approach and aligns with the projected impacts on the DeFi ecosystem.

Our future work will focus on two main areas: enhancing the solver algorithms and extending the cross-chain functionality. For the solver algorithms, we plan to incorporate more sophisticated decision-making capabilities that can dynamically adjust to varying market conditions. Additionally, we aim to broaden the cross-chain functionality to include more blockchains and improve the integration

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## **Authors' contribution**

Conceptualization: S.Z., Z.O.; methodology: S.Z., Z.O.; validation: S.Z.; investigation: S.S.; resources: S.S.; writing—original draft preparation: S.Z., S.S.; writing—review and editing: Z.O.; project administration: Z.O. All authors have read and agreed to the published version of the manuscript.

## **Conflicts of interests**

The authors declare that there are no conflicts of interest regarding the publication of this paper.

## References

- [1] Shi Z, Ivankovic V, Farshidi S, Surbiryala J, Zhou H, *et al.* AWESOME: an auction and witness enhanced SLA model for decentralized cloud marketplaces. *J. Cloud Comput.* 2022, 11(1):27.
- [2] Nakamoto S. Bitcoin: a peer-to-peer electronic cash system. 2008. Available: https://bitcoin.org/bitcoin.pdf (accessed: 2025–03–16).
- [3] Bergers J, Shi Z, Korsmit K, Zhao Z. Dwh-dim: a blockchain based decentralized integrity verification model for data warehouses. In 2021 IEEE International Conference on Blockchain (Blockchain), Melbourne, Australia, December 06–08, 2021, pp. 221–228.
- [4] Global projections 2023: blockchain utilization in finance sector. 2024. Available: https://www. reportlinker.com/p06244972/Blockchain-In-Banking-And-Financial-Services-Global-Market-Report.html?utm\_source=GNW (accessed: 2025–03–16).
- [5] Coinbase. 2024 crypto market outlook. 2023. Available: https://www.coinbase.com/nl/institut ional/research-insights/research/market-intelligence/2024-crypto-market-outlook (accessed: 2025–03–16).
- [6] Mohan V. Automated market makers and decentralized exchanges: a DeFi primer. *Financ. Innov.* 2022, 8:20.
- [7] Roth AE, Sönmez T, Ünver MU. Efficient kidney exchange: coincidence of wants in markets with compatibility-based preferences. *Am. Econ. Rev.* 2007, 97(3):828–851.
- [8] Coincidence of Wants. Available: https://docs.cow.fi/overview/coincidence-of-wants (accessed: 2023–11–6).
- [9] Shi Z, Farshidi S, Zhou H, Zhao Z. An auction and witness enhanced trustworthy sla model for decentralized cloud marketplaces. In *Proceedings of the Conference on Information Technology for Social Good*, Roma, Italy, September 9–11, 2021, pp. 109–114.
- [10] Ou W, Huang S, Zheng J, Zhang Q, Zeng G, *et al.* An overview on cross-chain: mechanism, platforms, challenges and advances. *Comput. Netw.* 2022, 218:109378.

- [11] Goes C. The interblockchain communication protocol: an overview. arXiv 2020, arXiv:2006.15918.
- [12] Goes C, Yin AS, Brink A. Anoma: a unified architecture for full-stack decentralised applications.
   2022. Available: https://github.com/anoma/whitepaper/blob/main/whitepaper.pdf (accessed: 2023–11–6).
- [13] CowSwap: First CoW Protocol UI. Available: https://github.com/cowprotocol/cowswap (accessed: 2023–11–6).
- [14] Foundation C. Picasso Network. Available: https://www.picasso.network/ (accessed: 2024–5–18).
- [15] Foundation C. MANTIS Whitepaper. 2024, pp. 1–16. Available: https://assets.website-files.c om/65b28e756a8eda2e91e76ca4/6656289f21123d6215091555\_MANTIS%20Whitepaper.pdf (accessed: 2024–5–18).
- [16] Composable Foundation. Available: https://www.composablefoundation.com/ (accessed: 2024–3–19).
- [17] Shi Z, Sweck S, Zaki O. From CoWs to multi-chain AMMs: a strategic optimization model for enhancing solvers. In 2024 IEEE International Conference on Blockchain (Blockchain), Copenhagen, Denmark, September 18, 2024, pp. 97–104.
- [18] Miller R. The MEVM, SUAVE Centauri, and beyond. 2023. Available: https://writings.flashbots.n et/mevm-suave-centauri-and-beyond (accessed: 2023–11–20).
- [19] Introduction: Cow Protocol. Available: https://docs.cow.fi/solvers/solvers (accessed: 2023–11–6).
- [20] Ramseyer G, Goyal M, Goel A, Mazières D. Augmenting batch exchanges with constant function market makers. In *Proceedings of the 25th ACM Conference on Economics and Computation*, New Haven, USA, July 8–11, 2024, pp. 986–1016.
- [21] Adams H, Zinsmeister N, Robinson D. Uniswap V2 Core. 2020, pp. 1–10. Available: https://uniswap.org/whitepaper.pdf (accessed: 2024–3–15).
- [22] Martinelli F, Mushegian N. A non-custodial portfolio manager, liquidity provider, and price sensor.
   2019, pp. 1–12. Available: https://docs.balancer.fi/whitepaper.pdf (accessed: 2024–3–15).
- [23] Belchior R, Vasconcelos A, Guerreiro S, Correia M. A survey on blockchain interoperability: past, present, and future trends. ACM Comput. Surv. 2021, 54(8):1–41.
- [24] Zheng J, Lee DKC, Quian D. An in-depth guide to cross-chain protocols under multi-chain world. World Sci. Annu. Rev. Fintech 2023, 1:2350003.
- [25] Walther T. Multi-token batch auctions with uniform clearing prices: features and models. 2018, pp. 1–19. Available: https://github.com/gnosis/dex-research/releases/download/v0.1.0/Multitoken.Batch.Auctions.with.Uniform.Clearing.Prices.pdf (accessed: 2023–10–18).
- [26] Angeris G, Evans A, Chitra T, Boyd S. Optimal routing for constant function market makers. In Proceedings of the 23rd ACM Conference on Economics and Computation, Boulder, USA, July 11–15, 2022, pp. 115–128.