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A comprehensive vulnerability assessment framework for water leakage in subway networks: case study of Beijing, China



Pengfei Li¹, Qing Xu^{1,*}, Yang Xiang¹, Yi Li¹, Junjie Zeng² and Sulei Zhang³

¹ Key Laboratory of Urban Security and Disaster Engineering, Ministry of Education, Beijing University of Technology, Beijing 100124, China

² UniSA STEM, University of South Australia, Adelaide 5095, Australia

³ School of Civil Engineering, Qingdao University of Technology, Qingdao 266033, China

* Corresponding author; E-mail: xqbjut@emails.bjut.edu.cn.

Highlights:

- Integrated framework combining network properties and leakage susceptibility is proposed.
- Highlight Fuzzy comprehensive evaluation is applied to quantify leakage uncertainty.
- Monte Carlo simulation is used to assess network robustness under failures.
- TOPSIS method is employed to prioritize critical stations.

Abstract: Water leakage in metro systems poses a persistent threat to structural durability and operational safety, particularly in water-bearing environments where leakage-induced deterioration may propagate through interconnected stations. While previous studies have extensively investigated leakage mechanisms and local structural responses, limited attention has been paid to the system-level vulnerability of metro networks under leakage disturbances, particularly the lack of an integrated framework linking leakage susceptibility and network characteristics. To address this gap, this study aims to develop an integrated framework for assessing station-level vulnerability in metro networks by incorporating both network characteristics and leakage susceptibility factors. Methodologically, network properties are first quantified by integrating topological structure and passenger flow characteristics. Leakage susceptibility is then evaluated using a fuzzy comprehensive evaluation method based on field investigation data. A combined weighting approach is further employed to integrate network and leakage indicators into a unified vulnerability assessment framework. Finally, a Monte Carlo probabilistic failure model is introduced to evaluate system robustness, and station vulnerability is ranked using the Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) method. The results indicate that leakage-related factors contribute dominantly to station vulnerability, accounting for approximately 62.4% of the overall weight, highlighting their critical role in metro system performance degradation. Several stations are identified as high-risk nodes due to the combined effects of unfavorable hydrogeological conditions and topological importance. The proposed framework can support infrastructure managers in prioritizing inspection scheduling, preventive maintenance, and targeted reinforcement, thereby



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enhancing the resilience of metro systems against leakage-induced disruptions. Future work should incorporate multi-temporal operational data and real-time monitoring information, and further validate the model using long-term maintenance records to improve its practical applicability.

Keywords: subway network; leakage susceptibility; probabilistic failure; station vulnerability; Monte Carlo simulation

1. Introduction

With the rapid development of subterranean infrastructure in China, the expansion of subway systems as efficient and convenient modes of public transportation has significantly improved people's lives [1,2]. Since the implementation of the 14th Five-Year Plan, China's subway system has progressively shifted from the construction-centric phase to a dual-phase focus on maintenance and construction [3,4]. This transition has imposed higher demands on the overall network operational stability and safety protection management capabilities. However, water leakage diseases in urban subway operations have been increasing annually, posing significant threats to both normal operations and structural safety [5–8]. When severe leakage diseases occur, they may induce disruptions to individual stations or even the entire subway network. In particular, water leakage incidents triggered by natural causes such as extreme rainfall frequently lead to station closures, severely compromising passenger travel safety and convenience.

Nowadays, studies on complex network vulnerability in urban rail transit primarily focus on the topological and functional characteristics of integrated systems, covering risk scenario identification, critical node evaluation, and resilience assessment under various disturbances. Yang *et al.* identified critical nodes in Beijing's subway network using a weighted neutralization method and analyzed vulnerability under random and targeted attacks based on GIS modeling [9]. Zhang *et al.* compared topological disparities among high-speed rail networks in China, the U.S., and Japan to reveal structural vulnerabilities under intentional disruptions [10]. To overcome the limitations of purely structural analyses, Li and Rong integrated functional metrics with topological indicators to assess network robustness [11]. Further extending this line of research, Wang *et al.* developed a two-layer network model incorporating infrastructure and service layers to simulate system degradation and recovery under natural disasters [12], while Zhao *et al.* proposed an agent-based flood resilience framework to evaluate cascading impacts on both network performance and passenger dynamics [13]. Zhang *et al.* expanded the scope to intermodal systems by analyzing functional losses in China's railway-air transport network using spatiotemporal modeling and multi-factor correlation analysis [14].

Meanwhile, several studies have extended traditional network theory to capture operational complexity. Noguchi and Fuse incorporated train reversal operations into vulnerability analysis to improve critical station identification [15]. Saadat *et al.* introduced a triangular efficiency-loss metric to quantify system resilience [16]. Martello *et al.* combined passenger-flow-weighted networks with recovery curves to assess resilience under sea-level rise scenarios [17], and Li *et al.* proposed a graph-theory-based method incorporating node weights [18]. In parallel, increasing attention has been paid to node criticality. Meng *et al.* developed a multi-attribute decision-making framework integrating weighted and unweighted indicators for node ranking [19]. Kopsidas and Kepaptsoglou further improved station criticality assessment by incorporating alternative services during disruptions [20]. Derrible and Kennedy introduced

the node clustering coefficient to characterize the Chicago subway system, linking topological structure with passenger flow dynamics [21].

Although complex network theory provides a macro-level perspective for understanding system vulnerability, research on water leakage in subway networks remains limited and lacks systematic theoretical integration. Existing studies have identified key drivers of leakage, including unfavorable geological conditions [4,22,23], construction defects such as inadequate joint sealing [6,24], and material degradation during long-term operation [25,26]. However, current assessments are largely confined to component-level or station-level analyses, and the network-level impacts of water leakage remain underexplored. This limitation can be attributed to three main challenges. First, most studies focus on sudden, large-scale disturbances, such as floods, earthquakes, and extreme weather events [27–29], while persistent and localized threats like leakage are rarely considered, particularly regarding their cascading effects on network topology and functional performance. Second, leakage-related data are often scarce and difficult to obtain, limiting empirical validation. Third, a unified framework integrating leakage characteristics with complex network theory is still lacking. Therefore, it is necessary to develop a systematic approach to assess subway network vulnerability under leakage disturbances, especially for large-scale systems such as the Beijing metro network.

To address these challenges, this study develops an integrated framework to assess station-level vulnerability in metro networks by incorporating both network characteristics and leakage susceptibility. The specific objectives are to: (i) construct a network indicator system integrating topological structure and passenger flow; (ii) evaluate leakage susceptibility using field data and a fuzzy comprehensive evaluation method; (iii) develop a unified vulnerability framework via a combined weighting approach; (iv) assess network robustness under leakage disturbances using a Monte Carlo-based probabilistic model; and (v) identify and prioritize critical stations using the Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) method. This study makes three main contributions. First, it establishes a coupling between localized leakage effects and network-level vulnerability propagation, bridging leakage studies and complex network theory. Second, it proposes a multi-source framework integrating structural, functional, and environmental factors. Third, it introduces probabilistic modeling to capture uncertainty in leakage-induced disturbances.

This study uses the Beijing subway network as a representative case. The analysis is based on available leakage records and static passenger flow data, and therefore does not account for temporal variations or extreme operational conditions. Nevertheless, the proposed framework provides a practical basis for vulnerability assessment and infrastructure management.

The remainder of this paper is organized as follows. Section 2 presents the methodology. Section 3 describes the network properties, including topological and functional characteristics, and establishes the multi-criteria framework. Section 4 evaluates leakage-related characteristics, including severity assessment, fuzzy comprehensive evaluation, and hydrogeological analysis. Section 5 assesses network vulnerability using a combined weighting approach, a Monte Carlo-based probabilistic model, and TOPSIS-based station prioritization. Section 6 concludes the study and outlines future research directions.

2. Methodology

2.1. Overview of the research framework

This section outlines a framework for assessing subway network vulnerability under leakage disturbances. It comprises four components: (i) network characterization based on topology and passenger flow; (ii) leakage susceptibility evaluation using fuzzy comprehensive evaluation; (iii) integration of network and leakage indicators through a combined weighting approach; and (iv) vulnerability assessment using a Monte Carlo-based probabilistic failure model with TOPSIS-based station prioritization. The overall workflow is illustrated in Figure 1.

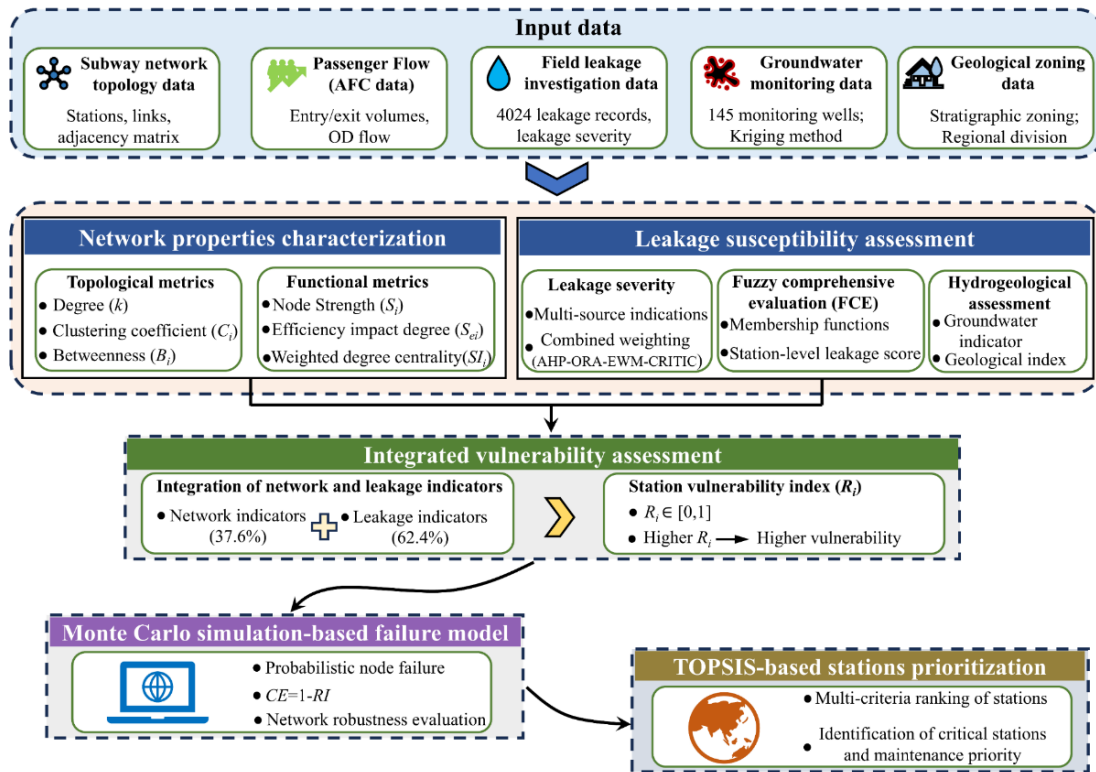


Figure 1. Overall framework of the proposed methodology.

2.2. Coupling mechanism between leakage and network vulnerability

Metro network vulnerability is governed by both structural characteristics and external disturbances. In this study, water leakage is treated as a localized yet persistent disturbance that degrades station performance and can propagate through the network. Physically, leakage-induced deterioration (e.g., structural damage, equipment malfunction, and service disruption) reduces station capacity. At the network level, stations function as critical nodes for passenger flow and connectivity; thus, node degradation may trigger cascading effects, including passenger redistribution, increased load on adjacent stations, and reduced network efficiency. Accordingly, leakage vulnerability is modeled as a node-level disturbance, while network vulnerability represents the system-level response. Their coupling is achieved by incorporating leakage susceptibility into the network assessment framework via weighted indicators and probabilistic failure modeling, enabling evaluation of how local leakage risks affect overall network performance and robustness.

3. Network properties of the subway system

Beijing's subway system (BSS) exhibits distinctive structural and functional characteristics. A six-indicator framework integrating topological and functional metrics is developed to evaluate network performance. The selected indicators are non-redundant and capture network resilience under disturbances, consistent with established multi-criteria approaches and validated in complex network studies [9,11].

3.1. Network topology characteristics

Based on the 2024 Beijing Metro, a Space L-based network is constructed with stations as nodes and adjacent connections as edges. Nodes are categorized as regular or transfer nodes, and the network is modeled as an undirected graph. As shown in Figure 2, the resulting network consists of 404 stations (Table 1) and 471 edges, capturing the topological structure of the BSS.

Table 1. BSS information and corresponding numbers.

Subway Line	No.	Station Name
Line 1	1–23	PGY, GC, BJYLY, BBS, YQL, WKS, WSL, GZF, JSBWG, MXD, NLSL, FXM, XD, TAMX, TAMD, WFJ, DD, JGM, YAL, GM, DWL, SH, SHD
Line 2	24–39	XZM, CGZ, FCM, ZCJ, XWM, HPM, QM, CWM, BJZ, ZYM, DSST, DZM, YHG, ADM, GLDJ, JST
Line 4	40–60	AHQB, BGM, XY, YMY, BJDSDM, ZGC, HDHZ, RMDX, WGC, GJTSG, DWY, XJK, PAL, XS, LJHT, CSK, TRT, BJNZ, MJB, JMX, GYXQ
Line 5	61–80	TTYB, TTY, TTYN, LSQ, LSQN, BYLB, DTL, HXXJBK, HXXJNK, HPXQ, HPLBJ, BXQ, ZZL, DS, DSK, CQK, TTDM, PHY, LJY, SJZ
Line 6	81–110	JAQ, YZ, XHC, LGZ, TC, HDWLJ, CSS, HYQ, BSQN, ELG, CGZX, BHB, NLGX, DDQ, HJL, JTL, SLP, QNL, DLP, HQ, CY, CF, WZXYL, TZBG, TYM, BYHX, BYHD, HJF, DXY, LC, BJXZ
Line 7	111–138	BJXZ, WZ, DGY, GAMN, HFQ, ZSK, QW, GQMN, GQMW, SJ, JLS, DJT, BZW, HG, NLZZ, HLGJQ, FT, SH, JHC, HC, LXZ, HZH, WSX, WSD, QF, GLJ, HZ, HQDJQ
Line 8	139–168	ZXZ, YZL, PXF, HLGDDJ, HY, YX, XXX, YTZ, LCQ, SLGYNM, ALPKGY, ATZX, BTC, AHQ, ADLBJ, SSH, ZGMSG, JYHT, TQ, YDMW, MXY, HHT, DHM, DHMN, HY, DGD, HJWY, WFT, DM, YH
Line 9	169–177	BDZ, LLQD, LLQ, QLZ, FTDDJ, FTNL, KYL, FTKJY, GGZ
Line 10	178–210	BG, SZJ, ZCL, ZCL, XTC, MDY, JDM, AZM, SYJ, TYG, SYQ, LMQ, NYZLG, TJH, JTXZ, JS, PJY, SLH, FZS, CSS, SLZ, JMD, CQ, JJM, SJM, FTZ, NW, XJ, LHQ, XDYT, CDG, CCQ, HQY
Line 11	211–213	MSK, BXA, XSG
Line 13	214–224	DZS, WDK, SD, QHZ, XEQ, LZ, HLG, BY, WJX, GXM, LF
Line 14	225–252	ZGZ, YBY, DWY, GZZ, DJ, DGT, LZSWQ, CHY, XTY, JFM, TRQ, JT, FZ, NBLZ, BGDXM, PLY, HM, ZYGY, ZY, DFBQ, JT, GJY, WJN, FT, WJ, DHQ, LGY, SGZ
Line 15	253–268	QHDLXK, LDK, BST, ALL, GZ, WJD, CGZ, MQY, SH, GZ, HLK, HSY, NFX, SM, SY, FB
Line 16	269–289	BAH, WYL, DXHL, TD, YF, YFN, XBW, MLW, NDNL, WQHQ, SZQ, WSS, GJK, YYTDM, HLNL, DGTN, FFQ, KD, YSZ, HTZ, WPC
Line 17	290–303	ZJZ, SBLD, BSS, CQB, CQ, JHH, GRTYC, ZJZ, XBH, HJY, QHY, TTYD, WLKXC, WLKXCB
Line 19	304–308	BTPZ, TPQ, NJ, XFD, XG
FangShan Line	309–321	HXDQ, BPY, DBT, DT, ZY, LBF, GYC, LXDxCB, LXDXC, LXDXCX, LXNG, SZ, YCD
ChangPing Line	322–335	CPXSK, SSLJQ, CP, CPDG, BSW, NS, SHGJY, SH, GHC, SMKXY, ZFB, QHXYQ, XZY, XYQ
YiZhuang Line	336–347	XC, XHM, JG, YZQ, YZWHY, WYJ, RJDI, RCDJ, TJNL, JHL, CQN, YZHCZ
YanFang Line	348–355	YS, FSCG, RYF, MGZ, DSHD, XC, YC, ZCW
S1 Line	356–361	SC, XY, LYZ, SA, QHY, SDQ
XiJiao Line	362–366	XS, ZWY, WA, CP, YHYXM
YiZhuang T1 Line	367–379	DHY, DHYX, JHYL, YCHZZX, YZTR, LQD, THL, JHC, SHZ, THQB, RHZ, RXJ, QZ
Capital Airport Line	380–381	No. 2 HZL, No. 3 HZL
Daxing Airport Line	382–383	DXXC, DXJC
BaTong Line	384–394	GBD, CMDX, SQ, GZ, BLQ, TZBY, GY, JKS, LY, LHL, TQ
DaXing Line	395–404	XHM, GMD, GMDN, ZY, QYL, HCXDJ, HCHCZ, YHZ, SWYYJD, TGY

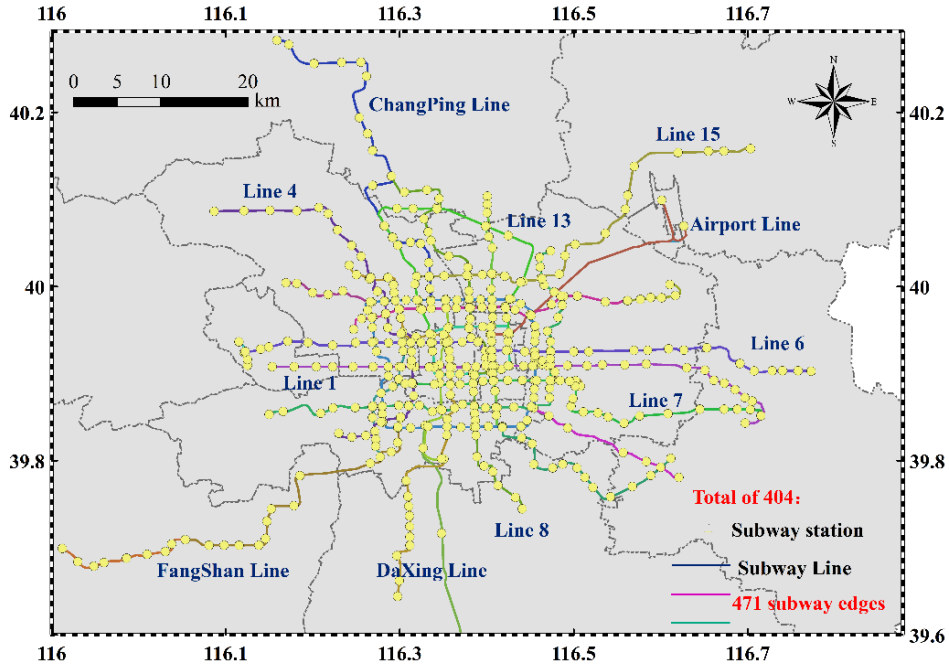


Figure 2. Topology of Beijing subway network.

3.1.1. Degree and degree distribution

The nodal degree in a metro network is defined as the number of edges incident to a station, reflecting its local connectivity. A higher degree indicates greater accessibility and operational importance. The degree of node k_i is formally defined as:

$$k_i = \sum_j a_{ij} \quad (1)$$

Where a_{ij} indicates the direct connectivity between stations i and j . The $a_{ij}=1$ if stations i and j are directly connected, otherwise $a_{ij}=0$.

The degree distribution $P(k)$, a fundamental topological descriptor, is defined as the probability that a randomly selected station possesses exactly k connections, calculated by:

$$P(k) = \frac{N_k}{N} \quad (2)$$

Where N_k denotes the number of stations with degree k , and N represents the total number of stations in the network.

The degree distribution of the Beijing subway network is shown in Figure 3. The majority of nodes exhibit a degree of 2, with a maximum observed degree of 6. A significant number of stations exhibit a degree value of 4, corresponding to major transfer stations. Nodes with degrees 1 and 2 account for approximately 80% of all nodes, indicating generally low local connectivity. Nevertheless, high-degree transfer nodes act as hubs that maintain overall network connectivity. This heterogeneous structure reflects a typical small-world property, implying robustness to random failures but vulnerability to targeted attacks.

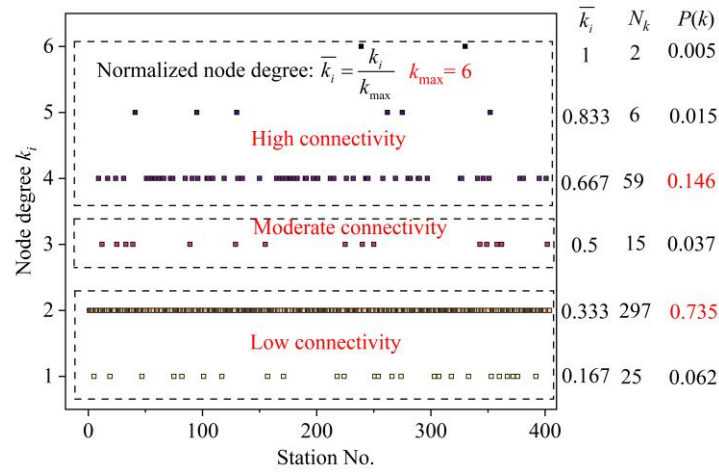


Figure 3. Degree distribution of Beijing subway network.

3.1.2. Station local clustering coefficient

The local clustering coefficient C_i quantifies the connectivity among the neighbors of node i . It is defined as the ratio of the number of existing edges n_i to the maximum possible number of edges among its k_i neighbors:

$$C_i = \begin{cases} \frac{2n_i}{k_i(k_i-1)}, & k_i > 1 \\ 0, & k_i = 1 \end{cases} \quad (3)$$

The coefficient C_i ranges from 0 to 1, with C_i close to 1 indicating a tightly interconnected cluster where the target station and its neighbors form a redundant subnetwork.

The clustering coefficients of 404 subway stations are plotted in Figure 4. Overall, more than 95% of stations exhibit a clustering coefficient of 0, indicating that their neighboring nodes are almost unconnected. A small portion of stations have clustering coefficients ranging from 0 to 0.4, suggesting a certain degree of connection among neighboring nodes. Notably, a few of stations reach a clustering coefficient of 1, corresponding to fully connected neighborhoods. Overall, the network exhibits weak clustering and a decentralized structure.

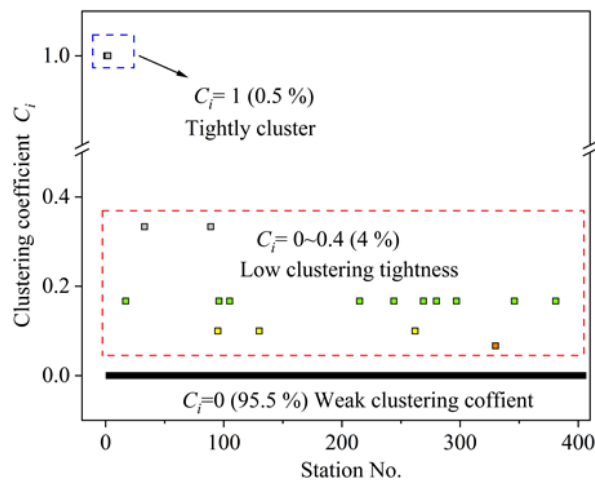


Figure 4. Clustering coefficients of Beijing subway network.

3.1.3. Betweenness

Betweenness B_i , a pivotal metric in the subway network, quantifies a station's topological influence by measuring its mediation capacity in global connectivity. It is defined as the proportion of shortest paths traversing the station i across all node pairs in the network:

$$B_i = \sum_{\substack{1 \leq j < l \leq N \\ j \neq i \neq l}} \left[\frac{n_{jl}(i)}{n_{jl}} \right] \quad (4)$$

Where $n_{jl}(i)$ represents the subset of these paths passing through station i , n_{jl} denotes the total number of shortest paths between stations j and l . Figure 5 illustrates the betweenness (B_i) distribution and normalized scatter plot of the Beijing subway network. The Beijing subway network displays a remarkable gradient-distribution feature, wherein the proximity to the center is directly correlated with elevated B_i values. This distribution indicates that core transfer-hub nodes perform a more significant transit role, whose betweenness values are notably higher than peripheral stations.

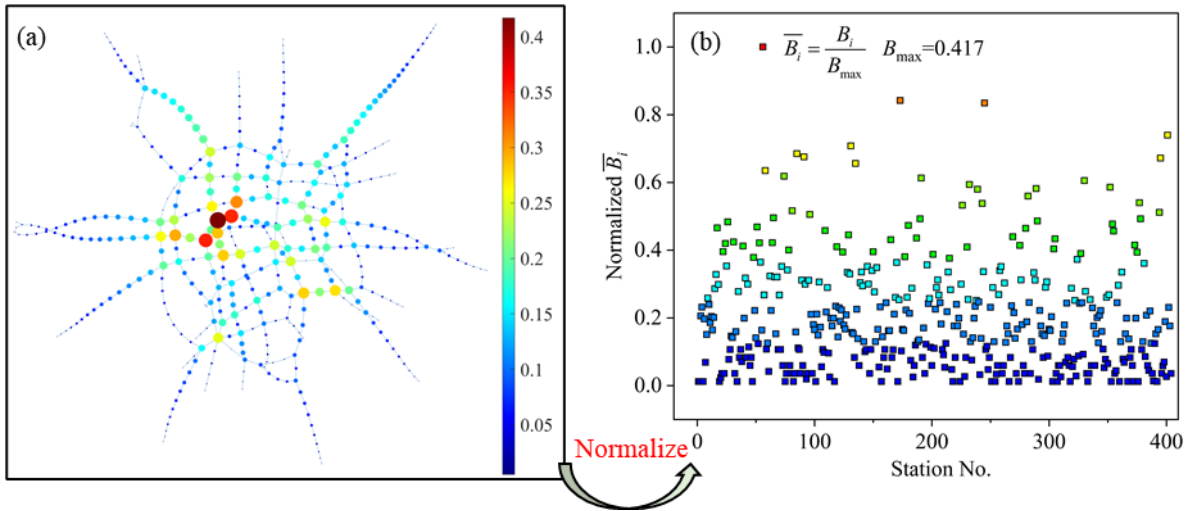


Figure 5. Beijing subway network betweenness distribution and normalized process.

3.2. Functional network characteristics

The functional network extends the topological network by incorporating operational dynamics, where nodes are weighted by passenger flow and edges by interstation distance. Metro system performance is characterized by both travel efficiency (e.g., travel time) and transport capacity (e.g., passenger volume). A data-driven approach is adopted to analyze the functional dynamics of the Beijing Subway System (BSS), using passenger flow data from a representative weekday (Wednesday, 26 June 2024). The dataset consists of 12,359,805 anonymized trips obtained from the Yikatong automatic fare collection (AFC) system, provided by the Beijing Mass Transit Railway Operation Corporation (BMTROC) under authorized access. Although the data are not available to the public, we obtained special permission to access the data (with personal identification removed) for this detailed analysis. Given the strong regularity of weekday passenger flow patterns, a typical weekday is considered sufficient to represent network dynamics [30]. Extreme scenarios (e.g., rainfall or holidays) are not included and will be addressed in future work.

3.2.1. Node Strength

In a weighted undirected network, the node strength S_i extends the concept of degree by incorporating edge weights. For station i , it is the sum of the weights of all edges directly connected. It can be expressed as:

$$S_i = \sum_j Q_{ij} \quad (5)$$

Where Q_{ij} is the passenger flow of the edge between stations i and j .

Figure 6 presents the normalized node strength values for the 404 subway stations. The maximum daily passenger flow occurs at BJNZ Station, reaching 226,782 passengers/day (normalized value = 1). As a transfer hub connecting multiple railway lines, this station also serves as a transfer node for subway Lines 4 and 14. Also as multi-line transfer hubs, the normalized node strengths of BJXZ (165,752 passengers) and CGZ (140,568 passengers) fall within the range of 0.4 to 0.8, forming a cluster of high-load nodes in the network.

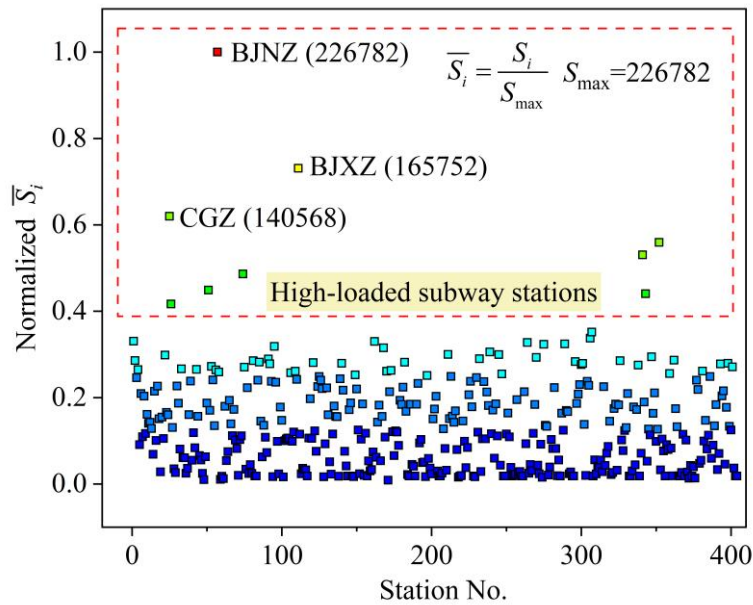


Figure 6. Normalized node strength for the 404 subway stations.

3.2.2. Station efficiency impact degree

To holistically evaluate metro network performance, a dual-criteria network efficiency metric E integrating travel time minimization and passenger flow maximization is proposed, which can be defined as:

$$E = \frac{1}{N(N-1)} \sum_{i \neq j} \frac{f_{ij}}{t_{ij}} \quad (6)$$

Where N is the total number of stations; f_{ij} is the daily passenger flow from station i to j ; and t_{ij} is the minimum travel time between i to j .

The OD passenger flow f_{ij} is derived from anonymized AFC records, capturing complete trip chains between origin and destination stations and thus reflecting actual travel demand.

The minimum travel time t_{ij} is calculated based on the official subway operation timetable, including in-vehicle time $t_{in-vehicle}$, platform waiting time t_{wait} , and a transfer penalties time $t_{transfer}$, denoted as follows:

$$t_{ij} = t_{in-vehicle} + t_{wait} + t_{transfer} \quad (7)$$

Where the in-vehicle time $t_{in-vehicle}$ and waiting time t_{wait} are determined from timetable data. A fixed transfer penalty of 5 minutes is adopted, which is consistent with typical transfer time assumptions in urban rail transit studies. The shortest travel path between station pairs are computed on the weighted network using these time components.

While the network efficiency metric evaluates subway system performance, it cannot directly assess individual station importance. To incorporate this measure into comprehensive evaluations, the station efficiency impact degree S_{ei} is proposed, which quantifies a station's marginal contribution to overall network efficiency:

$$S_{ei} = \frac{E_{max} - E_i}{E_{max}} \quad (8)$$

Where E_{max} is the maximum network efficiency; E_i is the network efficiency after removing station i and its adjacent edges. Figure 7 illustrates the distribution of normalized station efficiency impact degrees for the 404 subway stations. Approximately 97% of stations have values below 0.4, indicating that the removal of individual stations generally has limited impact on overall network performance and reflects high redundancy. In contrast, removing core multi-line stations significantly degrades network efficiency by extending the shortest paths, redistributing passenger flows, and prolonging commute times.

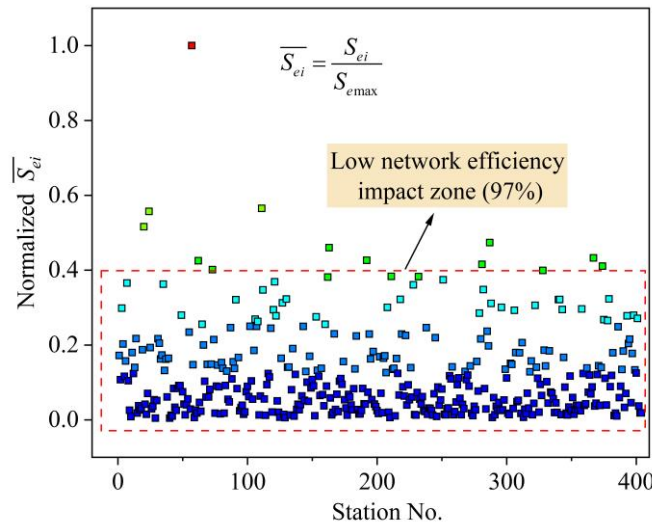


Figure 7. Normalized station efficiency impact degrees for the 404 subway stations.

3.2.3. Weighted degree centrality

In light of the covert progression of subway leakage and the difficulties in preemptive failure detection, a novel node criticality metric for weighted degree centrality SI is proposed. This metric integrates topological dominance and functional demand to optimize inspection resource allocation, which can be expressed as:

$$SI_i = \frac{k_i f_i}{k_{\max} f_{\max}} \quad (9)$$

Where k_i is the degree of station i ; f_i is the daily passenger flow of station i ; $k_{\max}$ and $f_{\max}$ are the maximum observed values across the network.

The normalized weighted degree centrality for the 404 subway stations is presented in Figure 8. Consistent with the S_i and S_{ei} , an exceedingly limited number of multi-line transfer hubs exhibit high SI_i values, thereby qualifying these nodes as critical control points within the subway network.

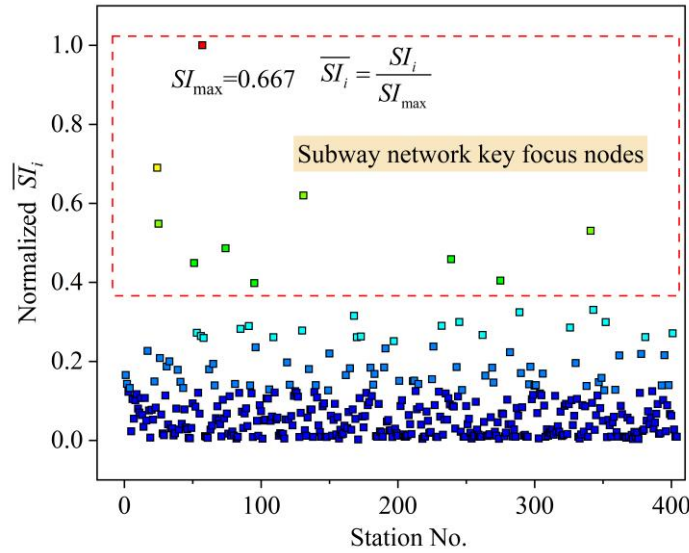


Figure 8. Normalized weighted degree centrality for the 404 subway stations.

3.3. Multi-criteria framework for metro network vulnerability

To holistically evaluate metro station importance, a multi-criteria framework combining topological (k_i , C_i , B_i) and functional metrics (S_i , S_{ei} , SI_i) was established. All six metrics are positive-directional quantitative indicators, enabling objective weighting through the Entropy Weight Method (EWM) [31] and the CRITIC Method [32]. The EWM has been demonstrated to quantify information uncertainty, with greater variability assigned to metrics for higher weights (lower entropy). It can be expressed as follows:

$$w_j^{EWM} = \frac{1 - H_j}{\sum_{m=1}^6 (1 - H_m)}, H_j = -\frac{1}{\ln N} \sum_{i=1}^N p_{ij} \ln p_{ij} \quad (10)$$

Where $p_{ij} = \frac{x_{ij}}{\sum x_{ij}}$, x_{ij} is the normalized metric value. The CRITIC method was calculated by combining the contrast intensity and conflict analysis, denoted as follows:

$$w_j^{CRITIC} = \frac{\sigma_j \sum_{k=1}^6 (1 - r_{jk})}{\sum_{m=1}^6 \left[\sigma_m \sum_{k=1}^6 (1 - r_{mk}) \right]} \quad (11)$$

Where σ_j is the standard deviation, r_{jk} is the Pearson correlation coefficient.

Based on Equations (10) and (11), the weights of BSS network characteristic indicators were calculated using the EWM and CRITIC methods, as presented in Table 2. The EWM yielded a relatively uniform weight distribution (ranging from 0.16 to 0.18), suggesting that each indicator provides a comparable informational contribution based on data dispersion. This reflects the intrinsic property of EWM, which primarily captures the variability of indicator values. Conversely, the CRITIC method produced a more pronounced differentiation in weighting, assigning higher significance to k_i (0.25) and S_i (0.24), while attenuating the influence of C_i (0.07). This variation arises because CRITIC simultaneously accounts for both the contrast intensity (standard deviation) and the conflicting character (inter-element correlation) among indicators, thereby accentuating metrics with superior discriminative capacity. These methodological discrepancies between EWM and CRITIC are complementary rather than contradictory, as they capture distinct statistical dimensions of the dataset. To integrate these complementary perspectives, a composite weighting approach was adopted by averaging the results of EWM and CRITIC. This strategy effectively balances dispersion-based and conflict-based information, mitigates potential bias from a single method, and enhances the robustness of weight determination.

Table 2. BSS network characteristic evaluation metric weights.

Metric	EWM	CRITIC	Composite Weight
Node degree k_i	0.16	0.25	0.205
Clustering coefficient C_i	0.18	0.07	0.125
Betweenness centrality B_i	0.16	0.12	0.14
Node strength S_i	0.16	0.24	0.20
Efficiency impact index S_{ei}	0.17	0.17	0.17
Weighted degree centrality SI_i	0.17	0.15	0.16

4. Leakage susceptibility evaluation of the BSS

To systematically address station failure risks induced by BSS leakage, it is imperative to establish a comprehensive evaluation system for leakage-inducing factors. The leakage metrics can be categorized into three groups: leakage severity, groundwater conditions, and geological factors. All stations were surveyed during a consistent seasonal period (April–June 2024) to ensure data comparability. Hydrogeological conditions were reviewed, and atypical sections were excluded. Regional groundwater characteristics were derived from long-term water-level data collected from 145 monitoring wells operated by the Beijing Municipal Water Authority. Leakage defects were measured using a tape measure (± 1 mm accuracy) to determine their extent and distribution. Crack widths were measured at least three times, and the modal value was adopted to reduce uncertainty on rough or moist surfaces. Leakage severity was classified into five levels (D1–D5) based on field observations and the Guide to Leakage Control Engineering Technology for Existing Urban Rail Transit Lines, with prior calibration and on-site verification conducted to ensure consistency.

4.1. Leakage severity evaluation model

4.1.1. Leakage severity classification

Based on field investigations for leakage in 404 subway stations of the Beijing subway, a total of 4024 leakage data points related to cracks, construction joints, and deformation joints were systematically obtained. Combining on-site surveys with characteristics of leakage level, impact degree, and leakage source attributes, a five-level leakage risk evaluation index was established, as presented in Table 3.

Table 3. Grading standards of evaluation indexes.

Leakage Type	Index	Leakage evaluation criteria				
		V_1	V_2	V_3	V_4	V_5
General	Leakage level	Infiltration	Drip	Line flow	Strand flow	Gush flow
	Impact degree	D	C	B	A	A-FS
Crack	Length (m)	< 5	5–10	10–15	15–20	> 20
	Width (mm)	< 0.5	0.5–0.8	0.8–1.0	1.0–1.2	> 1.2
Construction joint	Length (m)	< 5	5–10	10–15	15–20	> 20
	Width (mm)	< 0.5	0.5–0.8	0.8–1.0	1.0–1.2	> 1.2
Deformation joint	Length (m)	< 5	5–10	10–15	15–20	> 20
Structural surface	Area (m ²)	< 5	5–10	10–15	15–20	> 20
Hole	Number	1	2	3	4	> 4
Pipe root	Dimension (m)	< 1	1–2	2–3	3–4	> 4

4.1.2. Determination of combination weight

To evaluate station-level leakage severity, indicators from multiple leakage points are integrated and weighted. Scores are assigned to stations based on both quantity and severity, forming a unified station-scale evaluation system.

A hybrid weighting approach integrating subjective and objective methods is adopted for water leakage indicator weighting. Specifically, subjective weighting employs the Analytic Hierarchy Process (AHP) and Order Relation Analysis (ORA), while objective weighting utilizes the Entropy Weight Method (EWM) and Criteria Importance Through Intercriteria Correlation (CRITIC). To synthesize subjective and objective weights, importance coefficients are introduced to combine the four types of weights, enabling the calculation of optimal indicator weights through systematic integration.

For the subjective weights of indicators under each criterion layer, a set is established:

$$W_s = \{w_{sj} | 1 \leq s \leq 2, 1 \leq j \leq m\} \quad (12)$$

Where w_{sj} refers to the weight of the j -th factor under the s -th weighting method for a single criterion with m factors. For $\forall s$, it satisfies:

$$\sum_{j=1}^m w_{sj} = 1, w_{sj} \geq 0 \quad (13)$$

Similarly, for the objective weights, a set is established:

$$W_o = \{w_{oj} | 3 \leq o \leq 4, 1 \leq j \leq m\} \quad (14)$$

Where w_{oj} refers to the weight of the j -th factor under the o -th weighting method for a single criterion with m factors. For $\forall o$, it satisfies:

$$\sum_{j=1}^m w_{oj} = 1, w_{oj} \geq 0 \quad (15)$$

For each evaluation factor $z_j (1 \leq j \leq m)$, the subjective and objective importance α_j and β_j are calculated respectively through mathematical expectation. Taking the expectation of the sets of α_j and β_j , the subjective-objective importance coefficients α and β for the entire system are obtained. For the optimization model of combined weights, the linear weighted-sum method is employed. Through single-objective optimization for each evaluation factor, the combined weights can be derived as follows:

$$\begin{cases} \min H = \sum_{j=1}^m \alpha \sum_{s=1}^2 (w_j - w_{sj})^2 + \sum_{j=1}^m \beta \sum_{o=3}^4 (w_j - w_{oj})^2 \\ s.t. \sum_{j=1}^m w_j = 1 \\ 0 \leq w_j \leq 1 \\ 1 \leq j \leq m \end{cases} \quad (16)$$

Based on Equations (12)–(16), the importance coefficients of subjective and objective weights are obtained as $\alpha = 0.468$ and $\beta = 0.532$, respectively. Furthermore, the values of combined weights can be derived, as shown in Table 4.

Table 4. Calculation table for index weights of subway stations.

Leakage Type	Index	Subjective weights		Objective weights		Combined weights
		AHP	ORA	EWM	CRITIC	
Crack	Length	0.127	0.101	0.232	0.132	0.150
	Width	0.144	0.129	0.253	0.232	0.193
	Leakage level	0.431	0.559	0.424	0.348	0.437
	Impact degree	0.299	0.211	0.091	0.288	0.220
Construction joint	Length	0.103	0.088	0.198	0.063	0.114
	Width	0.117	0.112	0.279	0.268	0.199
	Leakage level	0.505	0.618	0.346	0.338	0.445
	Impact degree	0.274	0.182	0.177	0.330	0.242
Deformation joint	Length	0.125	0.099	0.067	0.124	0.103
	Leakage level	0.542	0.696	0.728	0.351	0.577
	Impact degree	0.333	0.205	0.205	0.524	0.320
Structural surface	Area	0.157	0.099	0.286	0.112	0.166
	Leakage level	0.49	0.696	0.507	0.503	0.546
	Impact degree	0.354	0.205	0.207	0.385	0.288
	Number	0.113	0.123	0.340	0.306	0.227
Hole	Leakage level	0.532	0.677	0.284	0.338	0.449
	Impact degree	0.354	0.2	0.376	0.356	0.324
	Dimension	0.174	0.144	0.622	0.270	0.312
Pipe root	Leakage level	0.463	0.622	0.232	0.276	0.389
	Impact degree	0.363	0.234	0.146	0.454	0.299

To evaluate the robustness of the weighting scheme against subjective bias, a sensitivity analysis was conducted. Subjective weights derived from AHP and ORA were perturbed by $\pm 10\%$ and $\pm 20\%$

to simulate variations in expert judgment. For each perturbation scenario, the combined weights were recalculated as follows:

$$w_{combined} = \alpha \cdot w_{subjective} + \beta \cdot w_{objective} \quad (17)$$

Where α and β are the importance coefficients derived by mathematical expectation, balancing the statistical distributions of subjective and objective weight sets.

Key indicators (e.g., leakage level, crack width) were selected to quantify the impact of perturbations. The deviation (D_{ev}) of combined weights from baseline values was calculated as:

$$D_{ev} = \frac{w_{perturbed} - w_{baseline}}{w_{baseline}} \times 100\% \quad (18)$$

The distribution of key indicator weights under different subjective weight fluctuations is plotted in Figure 9. The deviation of combined weights for key indicators from the baseline values is found to be less than 10% under $\pm 10\%$ perturbation, and less than 20% under $\pm 20\%$ perturbation. These results indicate that the combined weighting scheme is robust to moderate subjective variations. Although minor fluctuations in individual indicator weights are observed, the relative importance ranking of key indicators (e.g., leakage level and crack width) remains unchanged across all perturbation scenarios. Therefore, the proposed weighting approach demonstrates strong robustness and reliability for practical engineering applications.

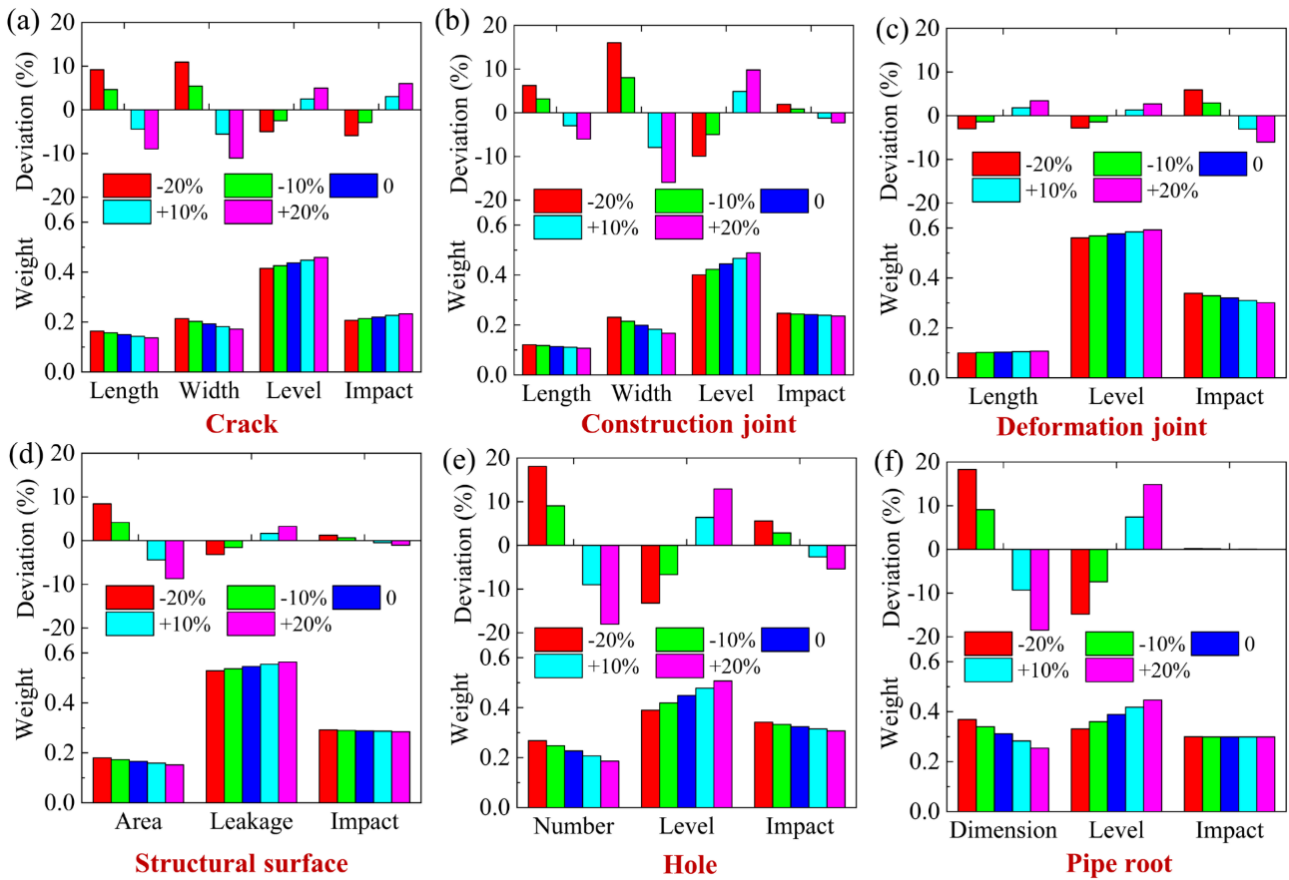


Figure 9. Distribution of key indicator weights under different subjective weight fluctuations.

4.2. Evaluation model based on the FCE method

The Fuzzy Comprehensive Evaluation (FCE) method is adopted to integrate 4024 leakage points across 404 stations into station-level vulnerability assessments. FCE enables a two-step process aligned with the data structure: (i) quantifying leakage attributes (e.g., severity) using fuzzy membership functions to address ambiguous boundaries, and (ii) aggregating point-level results into station-level indices through weighted summation. This approach preserves data granularity while enabling consistent station-level evaluation. In contrast, TOPSIS requires predefined station-level indicators (e.g., mean values), which may obscure important local variations. Machine learning methods typically require larger labeled datasets, and the limited sample size (404 stations) and imbalanced leakage distribution constrain their applicability. Therefore, FCE provides a more suitable and robust framework for this study.

4.2.1. Determination of membership function

In the evaluation of the leakage severity, selecting an appropriate membership function to assess leakage indicators is critical. Commonly used methods include trapezoidal distribution, Cauchy distribution, triangular distribution, normal distribution, *etc.* [33]. This study adopts a triangular distribution to construct the membership functions of the corresponding leakage indicators. Specifically, it mainly comprises three types: partial small type, middle type, and partial large type, as shown in Equations (19)–(21).

Based on the established five-level evaluation criteria, leakage indicators are categorized into continuous and discrete types. Continuous indicators such as crack length, width, area, and number permit direct quantitative processing through measurement standardization. Discrete indicators including leakage level and impact degree exhibit inherent fuzziness in detection results, requiring preliminary quantification through scoring: I (10), II (30), III (50), IV (80), and V (100). Guided by the criteria of the clearest point and fuzzy point aligned with the classification criteria described in Table 3, a series of membership functions is constructed as shown in Figure 10.

$$U_i(X) = \begin{cases} 1 & x \leq V_i \\ \frac{V_{i+1} - x}{V_{i+1} - V_i} & V_i < x \leq V_{i+1} \\ 0 & x > V_{i+1} \end{cases} \quad (i = 1) \quad (19)$$

$$U_i(X) = \begin{cases} 0 & x \leq V_{i-1}, x > V_{i+1} \\ \frac{x - V_{i-1}}{V_i - V_{i-1}} & V_{i-1} < x \leq V_i \\ \frac{V_{i+1} - x}{V_{i+1} - V_i} & V_i < x \leq V_{i+1} \end{cases} \quad (i = 2, 3, 4) \quad (20)$$

$$U_i(X) = \begin{cases} 0 & x \leq V_{i-1} \\ \frac{x - V_{i-1}}{V_i - V_{i-1}} & V_{i-1} < x \leq V_i \\ 1 & x > V_i \end{cases} \quad (i = 5) \quad (21)$$

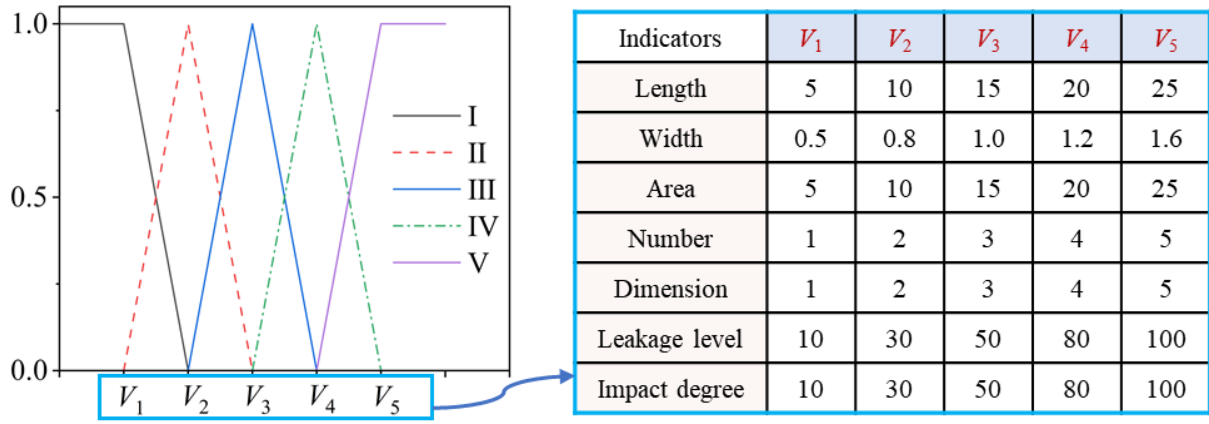


Figure 10. The distribution of membership functions for the leakage indicators.

4.2.2. Multi-criteria fuzzy comprehensive evaluation

Through membership functions calculation, the evaluation matrix R is constructed by systematically organizing evaluation sets comprising n evaluation indicators and their corresponding membership degree evaluations across m risk levels. The matrix is mathematically expressed as:

$$R = \begin{bmatrix} r_{11} & \cdots & r_{1m} \\ \vdots & \ddots & \vdots \\ r_{n1} & \cdots & r_{nm} \end{bmatrix} \quad (22)$$

Where r_{ij} denotes the membership degree of the i -th indicator to the j -th risk grade ($i = 1, 2, \dots, n; j = 1, 2, \dots, m$); Row vectors represent multi-criteria fuzzy evaluations for individual indicators; Column vectors characterize risk grade distributions across all indicators.

For each evaluation factor $z_j (1 \leq j \leq m)$, the factor weight vector is synthesized using the weighting method described in Section 3.1.2:

$$W_i = [w_1, w_2, \dots, w_n] \quad (23)$$

Through the fuzzy comprehensive evaluation (FCE) method, the $1 \times n$ -dimensional comprehensive evaluation matrix is derived as:

$$B = W \cdot R = [b_1, b_2, \dots, b_m] \quad (24)$$

Where W is the weight vector; R is the evaluation matrix.

The final leakage risk level is determined by applying the maximum membership principle to the matrix. Specifically, the risk level corresponding to the maximum value in B is selected as the evaluation result. When there are leakage points in subway stations, corresponding scoring criteria are assigned to the leakage risk levels of these points, as depicted in Table 5.

Table 5. Scoring criteria for each leakage risk level.

Leakage risk level	V_1	V_2	V_3	V_4	V_5
Score	1	2	4	7	10

Subsequently, the total leakage score for each station is calculated based on the number of leakages and their respective risk levels therein, expressed by the following:

$$SL_i = \sum_{j=1}^5 V_j n_{ij} \quad (25)$$

Where V_j denotes the score value for leakage risk level V_j , and n_{ij} represents the number of leakage points of risk level V_j in station i .

The normalized leakage scores of the 404 stations are presented in Figure 11. The ZSK and QXD stations have extremely high leakage indicators, significantly greater than the other stations.

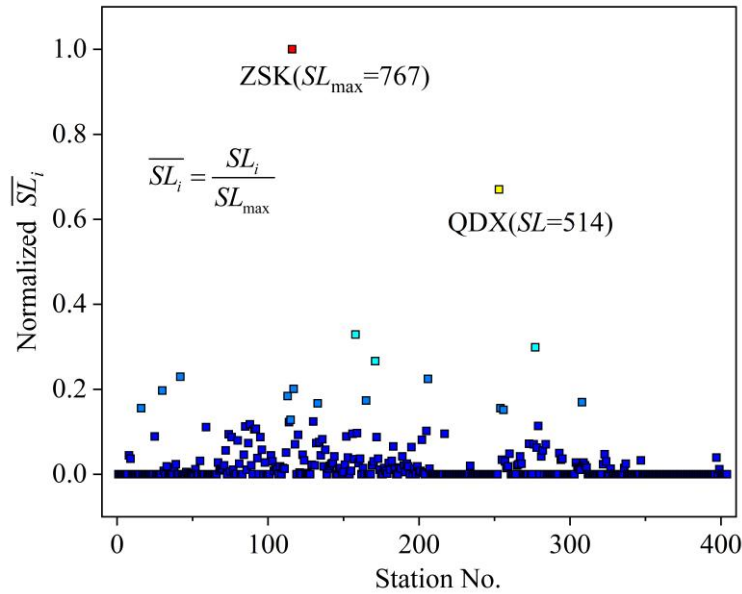


Figure 11. Normalized leakage scores of the 404 stations.

4.3. Hydrogeological environment evaluation model

4.3.1. Groundwater evaluation indicators

Groundwater depth data from 145 automated monitoring wells in Beijing were obtained from the Beijing Municipal Water Bureau. Using the ordinary Kriging method in ArcGIS, the distribution of groundwater depth in Beijing was generated (Figure 12). This enabled the calculation of interpolated groundwater levels at each subway station location. Subsequently, structural base slab depths at individual stations were acquired through field surveys. A normalized groundwater difference was calculated to derive the groundwater level indicator, which can be expressed as Equation (26). The Final normalized groundwater level indicator for the 404 subway stations is shown in (Figure 13). Fifteen subway stations, including LDK station and XY station, are situated in high groundwater level (exceeding 20 m), while the remaining stations are below 15 m.

$$I_{GW} = \frac{D_{slab} - D_{GW}}{\max(D_{slab} - D_{GW})} = \frac{H_i}{H_{\max}} \quad (26)$$

Where D_{slab} represents the base slab depth and D_{GW} denotes groundwater depth; H_i represents the difference between the base slab depth and groundwater depth at a specific station. Specifically, the indicator H_i is set to 0 when groundwater depth D_{GW} remains below the base slab depth D_{slab} .

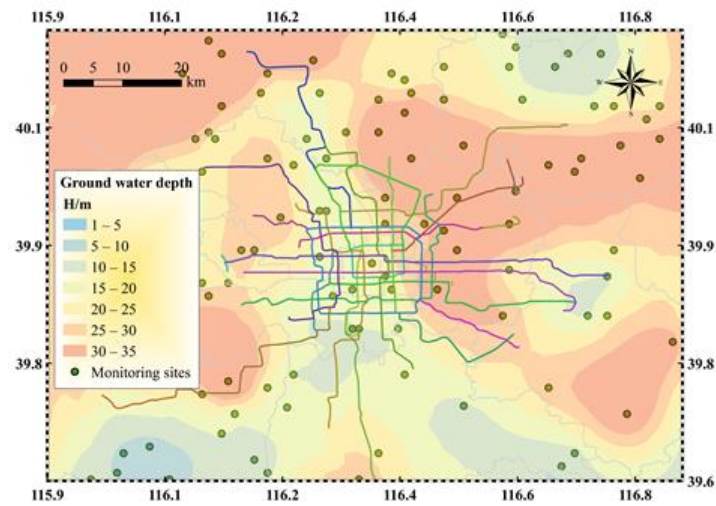


Figure 12. Distribution of groundwater depth in Beijing.

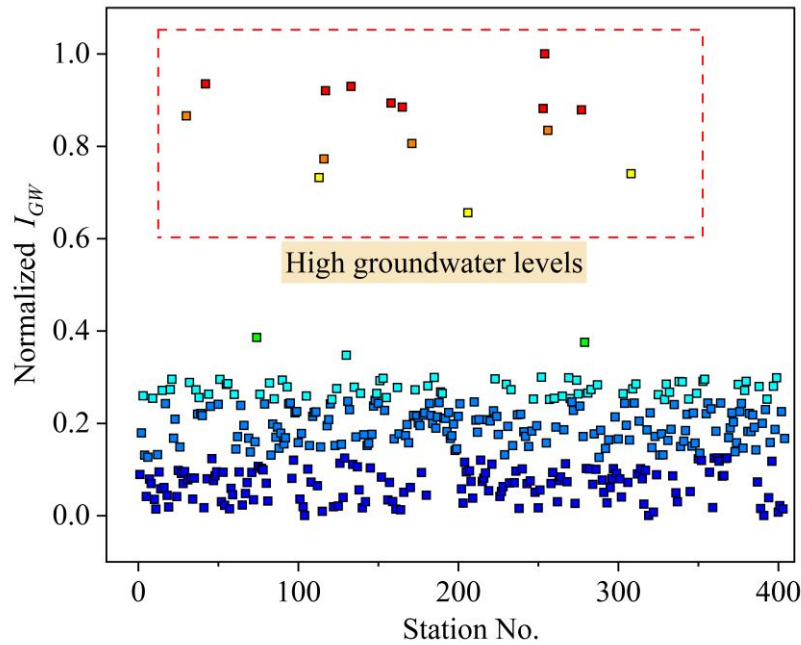


Figure 13. Normalized groundwater level indicator for the 404 subway stations.

4.3.2. Geological evaluation indicators

The Beijing subway area was classified into four distinct hydrogeological regions (A, B, C, D) using ArcGIS, as illustrated in Figure 14. Combined with the groundwater depth H_i data from Section 3.3.1, it can be determined that H_i in regions A and B mainly distributes in the range of 11–15 m, while that in regions C and D distributes across the range of 5–35 m. Stratigraphic zoning alone inadequately quantifies geological influences on subway leakage risks. Field investigations yielded 4024 validated leakage cases, which were statistically categorized by geological zones according to the FCE method. Results reveal significant spatial heterogeneity in leakage distribution and severity. Finally, the station-specific leakage scores within each zone were normalized to derive the geological index (Table 6). The most severe water leakage was observed in Region C, located in the northeastern part of central Beijing, followed by Region A in the southwest.

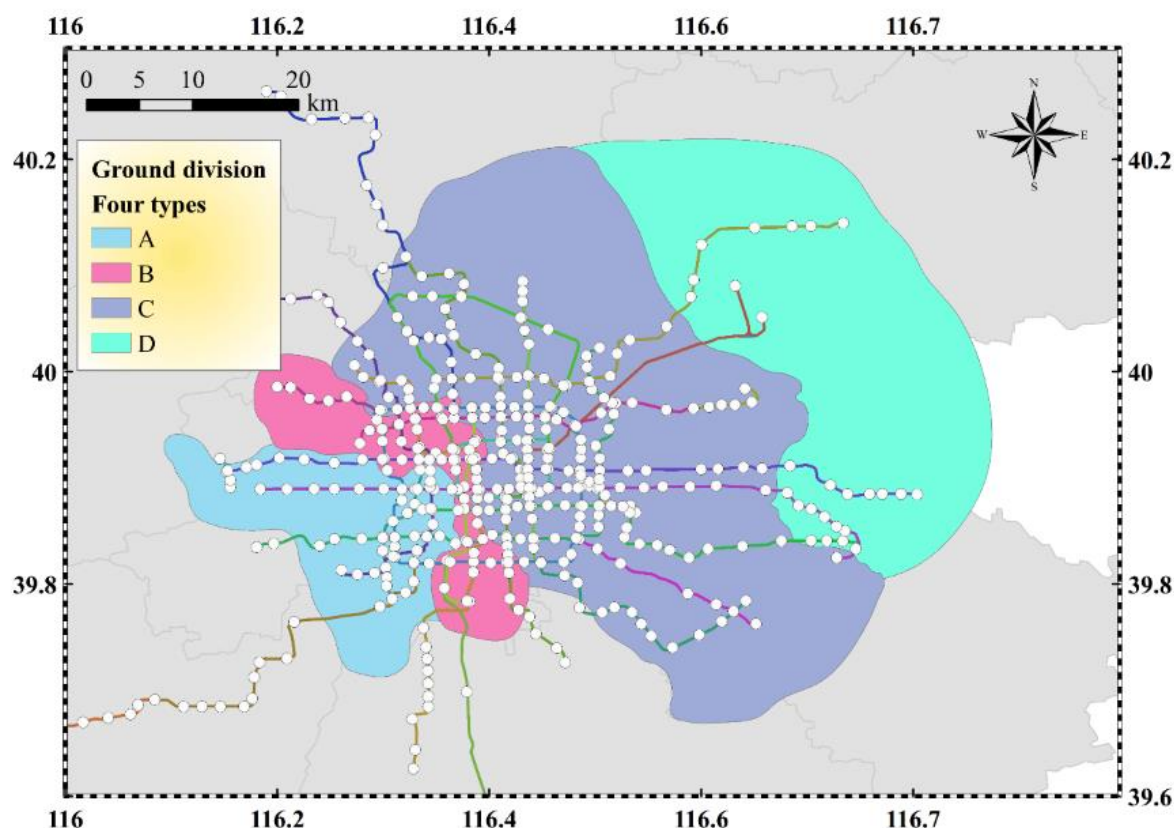


Figure 14. Beijing hydrogeological regions.

Table 6. Severity of subway station leakage by geological region.

Leakage region	Percentage of leaking stations	Average leakage number per station	Proportion of severe leakage points	Average leakage score	Geological index λ
A	49%	23.53	17.48%	61.88	0.94
B	45%	22.02	13.04%	44.06	0.67
C	46%	27.04	16.95%	65.81	1
D	56%	14.33	7.91%	27.65	0.42

5. Integrated vulnerability assessment of the Beijing subway network

5.1. Determination of comprehensive weights

Building upon the network properties (Section 3) and leakage susceptibility (Section 4), this section integrates the two components to conduct a comprehensive vulnerability assessment framework of the subway network. This framework comprises two secondary indicators: network properties (C1) and leakage susceptibility (C2), further decomposed into nine tertiary indicators (Figure 15). All contributing factors exhibit positive correlations with station vulnerability, reflecting the detrimental impacts on system performance.

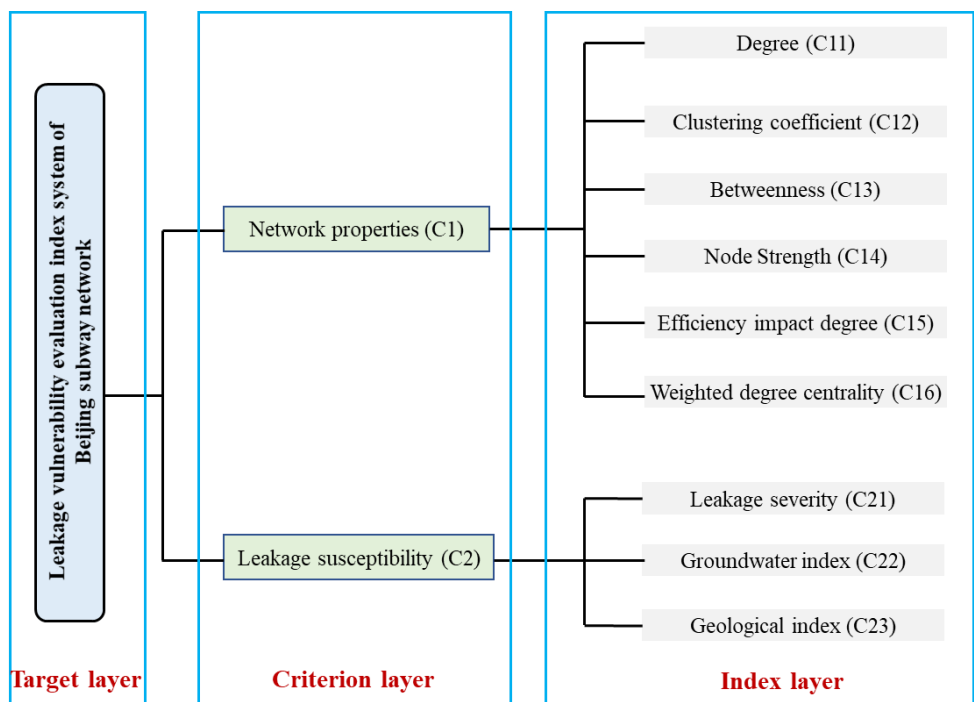


Figure 15. The leakage vulnerability evaluation index system of the Beijing subway network.

A total of 20 experts, comprising senior engineers from subway operation departments and academic researchers specializing in structural health monitoring, were invited to score and compare the importance of subway station vulnerability indicators. The comprehensive weights of each indicator are presented in Figure 16.

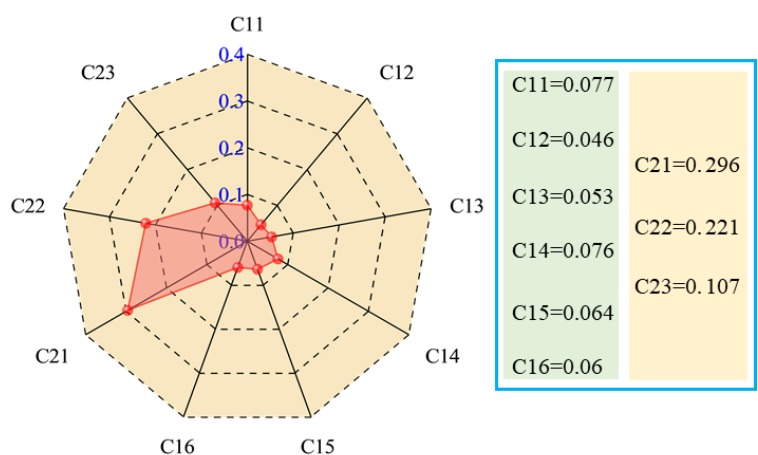


Figure 16. The calculated comprehensive weights of the evaluation indexes.

The weight results demonstrate that experts predominantly attribute greater significance to leakage indicators (totaling 62.4%) over network indicators (37.6%) in assessing station vulnerability. Among them, leakage severity (C21) receives the highest weight, reflecting its direct relevance to leakage conditions. The groundwater indicator (C22) is also assigned relatively high importance due to its role in preventive maintenance. In contrast, stratigraphic indicators contribute less to the overall evaluation. After integrating the comprehensive weights of all evaluation indicators, the vulnerability index (R_I) can be expressed as follows:

$$RI_i = C11 \cdot \bar{k}_i + C12 \cdot \bar{C}_i + C13 \cdot \bar{B}_i + C14 \cdot \bar{S}_i + C15 \cdot \bar{S}_{ei} + C16 \cdot \bar{S}L_i + C21 \cdot \bar{S}L_i + C22 \cdot \bar{I}_{GW_i} + C23 \cdot \bar{\lambda} \bar{S}L_i \quad (27)$$

RI ranges from 0 to 1, with values positively correlated to vulnerability: a higher RI indicates a station is more susceptible to leakage-induced disruptions. Finally, vulnerability index values (RI) for the 404 subway stations are presented in Figure 17.

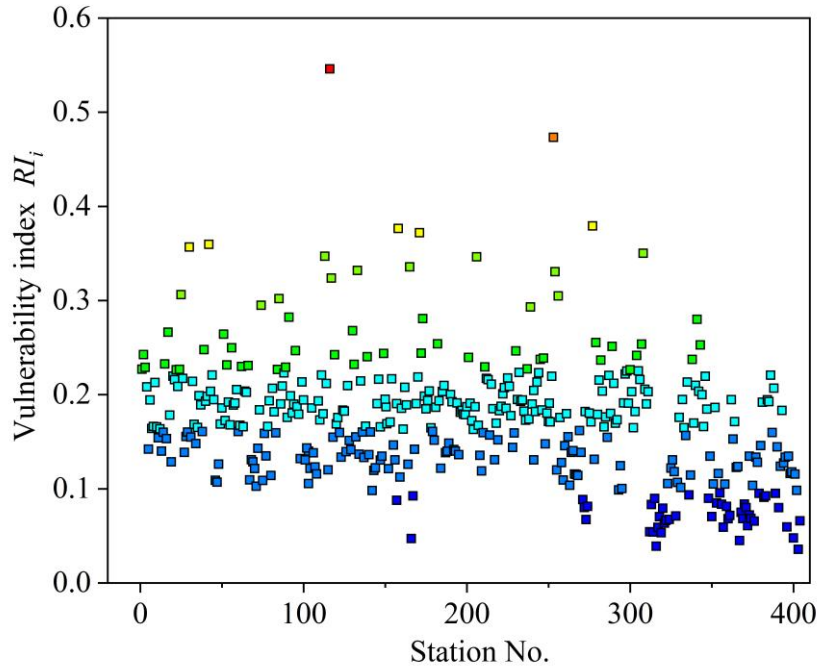


Figure 17. Vulnerability index values (RI) for the 404 subway stations.

5.2. Monte Carlo simulation-based probabilistic failure model

5.2.1. Vulnerability assessment framework

The vulnerability of the Beijing metro network is principally reflected in the diminished operational capacity of the metro nodes caused by leakage diseases. Previous studies have focused on reflecting the vulnerability of subway networks through random attacks [34] or deliberate attacks [35]. However, these approaches are not fully applicable to leakage-induced station failures. The deliberate attack model overlooks the randomness of failed nodes. Meanwhile, the random attack approach ignores the capacity tolerance of network nodes, meaning they may not be directly damaged when attacked. To address these limitations, a Monte Carlo simulation-based probabilistic failure model is proposed to quantify the impact of leakage on network vulnerability. The core objective of this method is to assess the important differences among stations in the entire network by simulating the probabilistic failure of stations due to water leakage. The Monte Carlo simulation of random attack flow is shown in Figure 18. Notably, the composite evaluation index (CE) is derived from the vulnerability index (RI) through the transformation $CE_i = 1 - RI_i$, which converts vulnerability into a safety-oriented metric. CE ranges from 0 to 1, where higher values indicate stronger resilience against leakage disturbances. This transformation enables the direct use of CE as the survival probability in Monte Carlo simulations, thereby ensuring a realistic representation of network performance degradation.

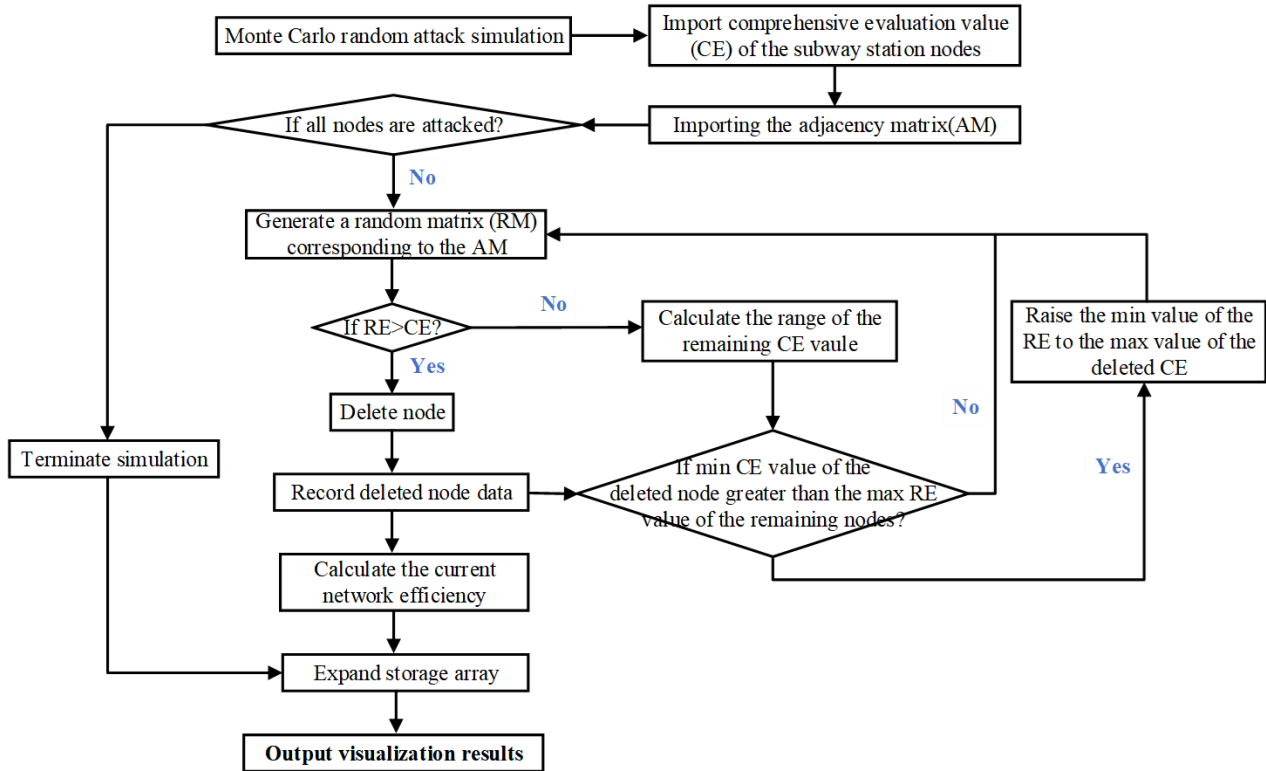


Figure 18. Monte Carlo simulation of random attack flow.

5.2.2. Result analysis

To validate the reliability of the Monte Carlo probabilistic failure model, simulations were conducted with 50, 300, and 1000 iterations, respectively. In each iteration, the number of randomly attacked nodes was set within the range [1, 404], encompassing all scenarios from isolated node failures to full network collapse. The dynamic evolution of the network robustness of subway operations under different attack types is illustrated in Figure 19. It can be seen that after more than 300 iterations, the network robustness distribution diagram has basically converged, revealing consistent trends in the three-stage robustness evolution. Therefore, Monte Carlo simulations were performed with 1000 independent iterations to meet the accuracy requirements of the results. Both Monte Carlo and random attack scenarios demonstrated significantly higher robustness compared to deliberate attacks. Furthermore, the Monte Carlo random attacks outperforms traditional random attacks in capturing stochastic network vulnerabilities, although both methodologies yield comparable overall trends. During the initial Stage-I (< 20% node removal), the network robustness distributions under both stochastic modes exhibited low variance with gradual decline, attributed to preserved connectivity through critical hub retention. In Stage-II (20%–50% node removal), robustness dispersion increases markedly, reflecting enhanced heterogeneity in nodal impacts as disruptions intensify. The Monte Carlo method exhibited greater variance due to its probabilistic failure thresholds, which combined stochastic factors and nodal vulnerability. In this manner, cumulative disruption effects amplified robustness fluctuations. When more than node removal exceeds (Stage-III), the network undergoes structural collapse, the network undergoes structural collapse, and the residual robustness stabilizes near its theoretical minimum. In summary, the disruption of critical nodes significantly deteriorates the overall network robustness. Therefore, leakage prevention and control should prioritize key station nodes.

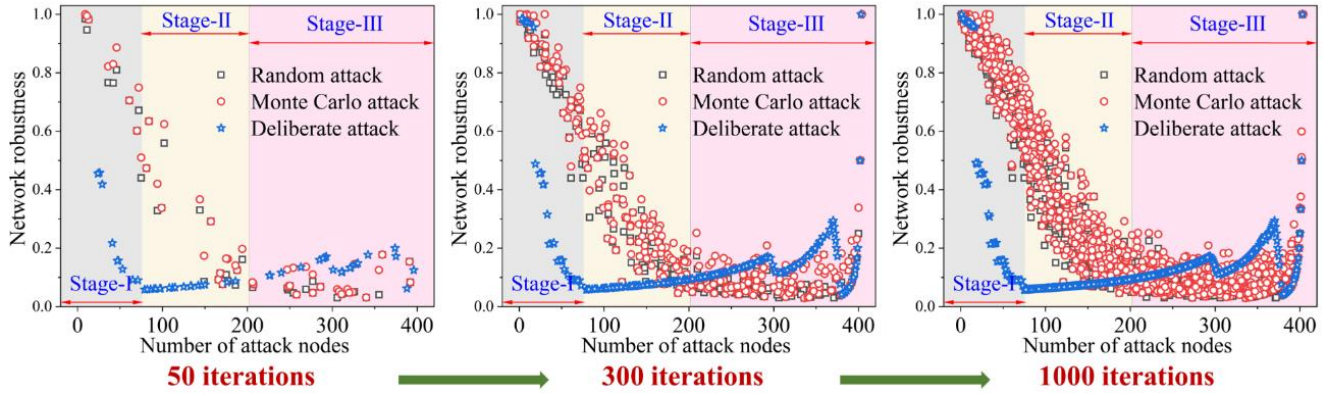


Figure 19. Impact of the three attacks on network robustness.

5.3. TOPSIS-based prioritization of critical Stations

5.3.1. Critical subway station identification

To quantify the real impact of individual nodes on network vulnerability and identify critical nodes, this study evaluates node influence under varying levels of network disruption. Specifically, changes in network robustness before and after node removal are measured at three key stages of Monte Carlo simulations. A larger robustness variation indicates higher node criticality. The TOPSIS method is adopted as the analytical framework to rank multi-criteria ideal solutions for station nodes.

In Monte Carlo random attacks, the pre-damage and post-damage network robustness changes for each damaged station are extracted at three distinct stages (0%–20%, 20%–50%, 50%–100%) to quantify its impact. Without additional weight distinction, each index is normalized to construct the initial matrix Z . To identify the positive ideal solution z^+ (composed of each column's maximum element) and the negative ideal solution z^- (composed of each column's minimum element), it is necessary to consider the matrix Z . The distance of each evaluation index from the positive and negative ideal solutions is calculated as follows:

$$D_i^+ = \sqrt{\sum_{j=1}^m (z_j^+ - z_{ij})^2}, D_i^- = \sqrt{\sum_{j=1}^m (z_j^- - z_{ij})^2} \quad (28)$$

Subsequently, the closeness C_i of each station to the positive ideal solution is computed. The closer the value is to 1, the higher the importance of the station. It can be expressed as:

$$C_i = \frac{D_i^-}{D_i^+ + D_i^-} \quad (29)$$

Figure 20 illustrates the top 20 stations ranked by importance using TOPSIS. These station disruptions can substantially impair the robustness of the subway network. ZSK Station and QDX Station exhibit significantly elevated importance (0.84 and 0.73, respectively), attributed to their roles as high-capacity transit hubs with high daily ridership, high groundwater levels, and histories of severe leakage. Consequently, they possess extremely high vulnerability in the network and serve as key nodes at all stages. Most top 20 stations are transfer stations clustered in the network core. Additionally, some non-transfer stations with low topological scores but high leakage indices exhibit elevated vulnerability and importance.

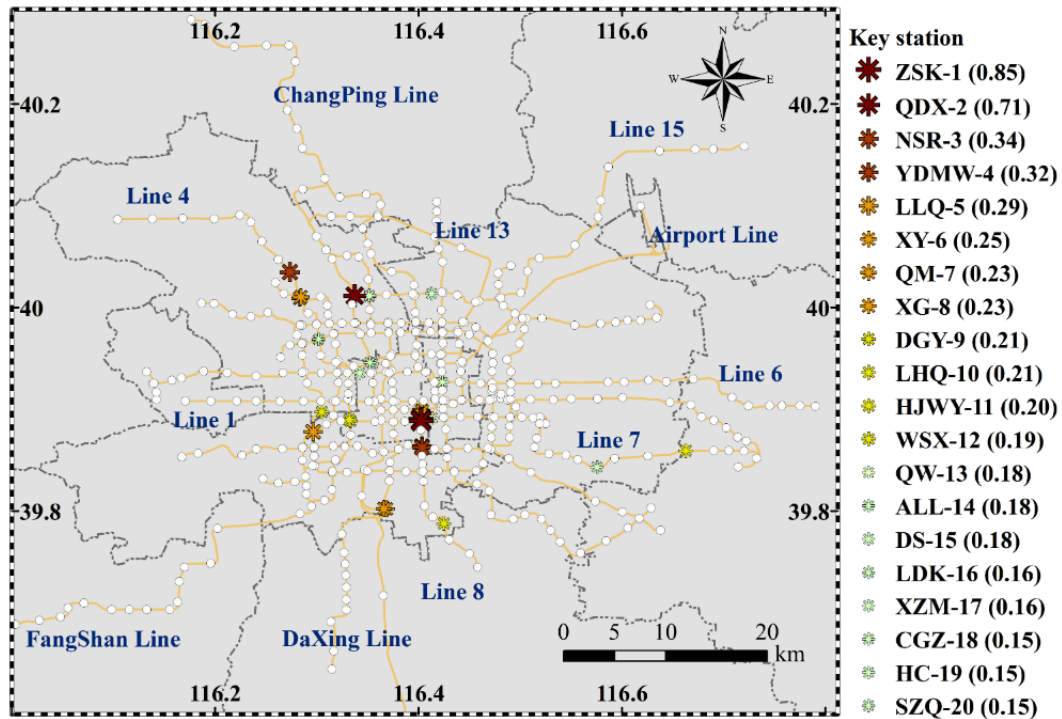


Figure 20. Top 20 sites of importance.

5.3.2. Case analysis and empirical validation

The TOPSIS results identify ZSK (Zhushikou) and QDX (Qinghuadonglu Xikou) as the most vulnerable stations. To externally validate these findings, their geographic settings, structural characteristics, and maintenance records were further examined.

Located in the central axis area of Beijing (Dongcheng District), ZSK serves as a critical interchange between Lines 7 and 8. Its high vulnerability score (0.84) arises from the combined effects of network importance and structural susceptibility. The station features a deep, multi-level underground structure in Region C (Figure 14), where complex hydrogeological conditions and high hydrostatic pressure (20–25 m, Figure 12) prevail. Field surveys identified 364 leakage points, mainly at structural joints and deformation gaps. This high defect density is consistent with frequent water-plugging grouting operations over the past three years, confirming that the interaction between network centrality and leakage susceptibility significantly increases operational risk.

As the terminal of Line 15 (Haidian District), QDX exhibits a distinct vulnerability profile, with a score of 0.73. Located near a groundwater recharge zone, the station is subject to persistent seepage from surrounding sandy gravel strata. Field inspections recorded 250 leakage points. Despite not being an interchange station, QDX ranks highly due to the dominant contribution of leakage-related indicators (62.4%). This result is consistent with expert judgment emphasizing structural condition in operational safety and aligns with its classification as a “high-maintenance” station in operational records.

The strong agreement between model-identified high-risk stations and historically recognized problem sites provides robust external validation of the proposed framework. It demonstrates the model’s capability to distinguish vulnerability mechanisms driven by network topology, hydrogeological conditions, or their combined effects.

5.3.3. Engineering application guidelines

Based on the TOPSIS results, station importance is quantified by the closeness coefficient (C_i). To translate these rankings into practical risk control strategies, a three-level risk management framework aligned with the importance index (C_i) is proposed, as shown in Figure 21. It can be seen that the priority of station treatment can be divided into high-critical stations, moderate-critical stations, and low-critical stations. As station criticality increases, both monitoring frequency and warning intensity should be correspondingly enhanced. Moderate-critical stations require biweekly inspections, while low-critical stations can be managed with quarterly inspections. Mitigation strategies also vary across risk levels. High-critical stations require immediate leakage remediation and contingency planning. Moderate-critical stations should adopt targeted repair and preventive maintenance measures. For low-critical stations, routine preventive maintenance is generally sufficient. The strategy provides a scientific decision-making basis for subway risk prevention and control.

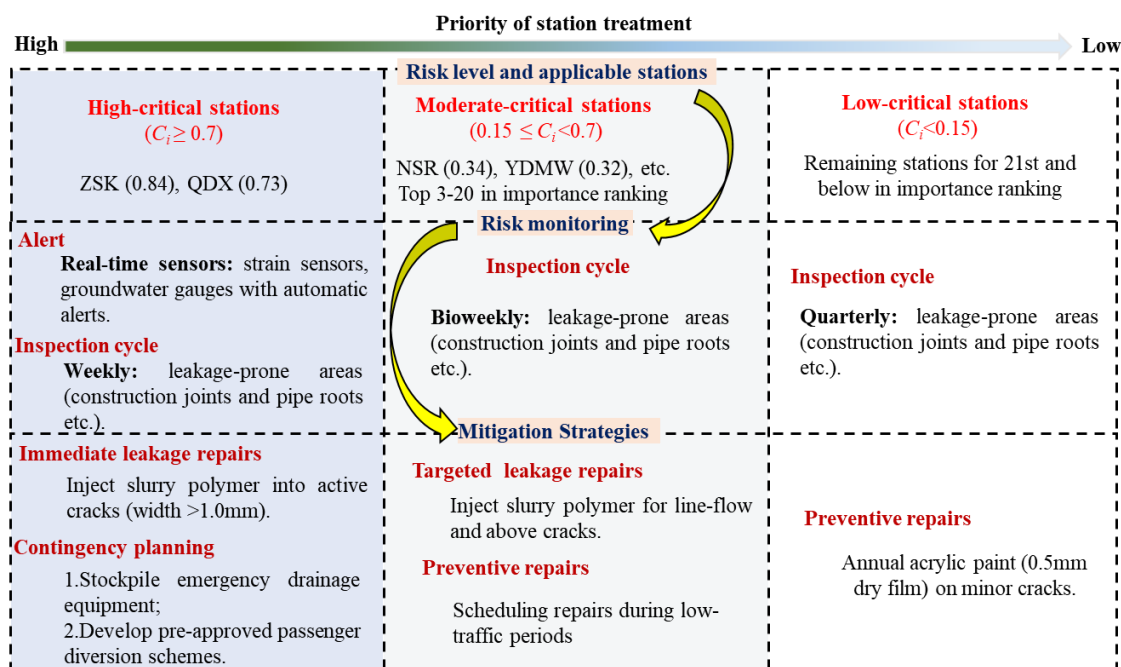


Figure 21. A three-level risk management framework in engineering application.

6. Conclusion

Severe leakage can disrupt individual stations and compromise overall subway network performance. This paper proposes a comprehensive vulnerability assessment method for integrating the subway network property and leakage susceptibility to evaluate subway operational safety. The main conclusions are as follows:

- (1) The Beijing subway network is characterized using six indicators, including topological metrics (degree, clustering coefficient, and betweenness) and functional metrics (node strength, station efficiency impact, and weighted degree centrality). A combined EWM-CRITIC weighting scheme identifies degree (0.205) and node strength (0.20) as the most influential indicators, highlighting the critical role of hub stations in maintaining connectivity and passenger flow capacity.

(2) The fuzzy comprehensive evaluation method is applied to rate leakage risk level for 4024 leakage points. Station-level leakage severity is derived by aggregating leakage quantity and risk levels, identifying ZSK and QDX as the most critical stations. Groundwater conditions, estimated via ordinary Kriging, indicate that 15 stations (e.g., LDK and XY) are located in high groundwater environments. Spatial analysis further reveals that Region C (northeast Beijing) exhibits the highest leakage severity, followed by Region A (southwest).

(3) By integrating network and leakage indicators, a comprehensive vulnerability model is established. Leakage-related factors account for 62.4% of the total weight, significantly exceeding network factors (37.6%), underscoring the dominant role of leakage conditions in determining network vulnerability.

(4) To address the limitations of traditional attack models, the Monte Carlo probabilistic failure model was introduced to identify critical stations. Monte Carlo simulations of network failure processes exhibit three-stage characteristics: stability (Stage-I), abrupt change (Stage-II), and re-stabilization (Stage-III). When over 50% of nodes are attacked (Stage-III), network robustness approaches collapse, validating the necessity of the key station priority protection strategy.

(5) The TOPSIS method was applied to assess station importance, identifying ZSK station and QDX station as the most vulnerable nodes in the entire network due to their transfer functions, high passenger flow density, and leakage histories. A three-level risk management framework aligned with the importance index (C_i) is proposed for engineering priority management and monitoring.

The proposed method provides practical support for infrastructure managers to identify critical stations and optimize maintenance prioritization. However, this study is subject to several limitations. The analysis is based on data from a single urban network, and certain model parameters rely on empirical assumptions, which may introduce uncertainty. In addition, the temporal evolution of leakage and dynamic operational conditions are not explicitly considered. Future research will focus on incorporating time-dependent data, improving model generalizability across different cities, and integrating advanced data-driven techniques to enhance prediction accuracy and real-time decision-making capability.

Data availability declaration

No supplementary or additional data were generated in this study.

Declaration of generative AI and AI-assisted technologies

During the preparation of this manuscript, the authors used ChatGPT only for language polishing and readability improvement. The research conception, methodology, analysis, interpretation of results, and scientific innovations were independently completed by the authors. The authors take full responsibility for the content of the manuscript.

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Authors' contribution

Conceptualization, Pengfei Li and Qing Xu; methodology, Pengfei Li; software, Qing Xu; validation, Yi Li, Junjie Zeng and Sulei Zhang; formal analysis, Qing Xu; investigation, Yang Xiang; resources, Pengfei Li; data curation, Yang Xiang; writing—original draft preparation, Qing Xu; writing—review and editing, Pengfei Li; visualization, Sulei Zhang; supervision, Pengfei Li; project administration, Pengfei Li; funding acquisition, Pengfei Li. All authors have read and agreed to the published version of the manuscript.

Conflicts of interest

The authors declare no competing interests.

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